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Resurrecting the Original Intent from Old Cadastral and Engineering Data

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ABSTRACT

In most cases, cadastral surveyors need to start their field activities and their subsequent office calculations by re-establishing previous geometries, often without the aid of the previous field measurements, but simply the resultant geometries of the previous surveyor(s). The previous geometries are often represented by a series of dimensions, both length and angle/bearing, on survey plans showing rounded numbers which may or may not follow a rounding convention. If a rounding has been used, it is rarely stated or recorded. To solve this issue, several methods have been used. Most of these methods consider each type of measurement in a particular order, starting with the first, then moving on to the next. This paper documents the thoughts and efforts to derive a more complete solution, where every dimension is considered on its merits, including implied dimensions which can sometimes be a little hidden. The paper includes the use of different methods to come up with solutions, including methods used by Albert Einstein (although not mastered by the author) and more familiar methods like least squares approaches. Potential applications of the methods are mentioned, although not all have been developed.

KEYWORDS: *Geometry, alignment, re-establishing intent, least squares, cadastral.*

1 INTRODUCTION

In the past, many organisations and authorities have stored records and configuration information in paper-based systems. In cadastral situations, this information has been stored in deeds, sketches and plans held in one of several different authority plan rooms, including the Registrar General's office, Crown Lands office, Department of Mining and Mineral Resources, NSW Railways, government gazettes and perhaps others. In the engineering space, there is an even a wider range of locations that have held plans for designs. Some records originated in private holdings and were delivered to these locations as required in the circumstances.

In the majority of cases, the calculations were done to a higher level of precision than that which is shown in the documents (with rounding to a millimetre and in some cases larger values). This rounding may have been due to regulation, industry practice, individual assignment of the appropriate accuracies of the dimensions, or perhaps by contract requirements to meet standards and specifications.

In the past, when further work was required (usually improvements in geometry or where additional infrastructure was required to be installed), initial searches would be undertaken with the result being a quantity of paper-based records (including images of documents in this category), and more recently sometimes Computer-Aided Design (CAD) or xml-based documents. The CAD and xml documents may or may not include the rounding, depending on the processes used to publish them. The problem is that engineering design has transitioned to a computerised environment and the rounded data has not fitted together in the way that was expected by the software for the ongoing task.

This problem is often ignored in the first instance and then avoided or hidden as the task goes on. Although avoiding or hiding the problem may be appropriate in some circumstances, there are also times where it would be much better to solve the problem up front to make the ongoing calculations much smoother.

This paper documents the author's thoughts and efforts to derive a solution, where every dimension is considered on its merits, including implied dimensions which can sometimes be a little hidden. It includes the use of different methods to come up with solutions, including methods used by Albert Einstein (although not mastered by the author) and more familiar methods like least squares approaches. Potential applications of the methods are mentioned, although not all have been developed.

2 PONDERING THE PROBLEM WHILE HIDING IT FROM THE TASK

Over several years, the author has had times of thinking through the problem and trying to come up with a solution. Based on the author's background, the scenarios considered are mostly railway alignment type scenarios. These scenarios have application in both cadastral and engineering cases in the railway environment, as many railway cadastral boundaries are defined based on the railway alignment.

The thoughts around how to regenerate alignments shown on plans can result in several possibilities. These include starting with arcs because they have long distances from the centre to the alignment (driven by the training of working from the whole to the part), but this breaks down when there are long straights with tight curves on each end. Traditionally, alignment reconstruction would start with intersection points, although often these are not documented on the plans as they can be out of the sheet plot area.

Much time has been spent trying to develop a combination of these methods where the curve radii lengths are compared to the lengths of the straights to decide which to choose. Developing this method raises additional problems in developing algorithms that can cater for the general case.

During this time of thinking that 'there must be a better way', the author was given a copy of Albert Einstein's biography by Walter Isaacson (2007). The author was taken by the possibility of deriving solutions using only thought experiments. At the time, the standout solution was taken by the development of a geographic proof of Pythagoras's Theorem, which is now taught to high school students.

In Isaacson (2007), a description of Einstein's thought experiments is given several times. The common thread is that as Einstein was developing his ideas, he would test them by the only way available to him. He would conceive a situation relevant to the theme of his work, then develop this situation using what he already knew, stepping through the stages until he came to the point that would either support or refute his idea. On page 114 (Isaacson, 2007), there is a quote from Einstein about a thought experiment that he undertook when he was 16 that led eventually to his theory of Special Relativity. Of course, some of his ideas were also supported later by practical observations which were not available at the time.

Unfortunately, the author has had many attempts at 'thought experiments' about the topic of this paper without being able to bring these to a conclusion. In the end, trial and error experiments have been the main method of developing this paper.

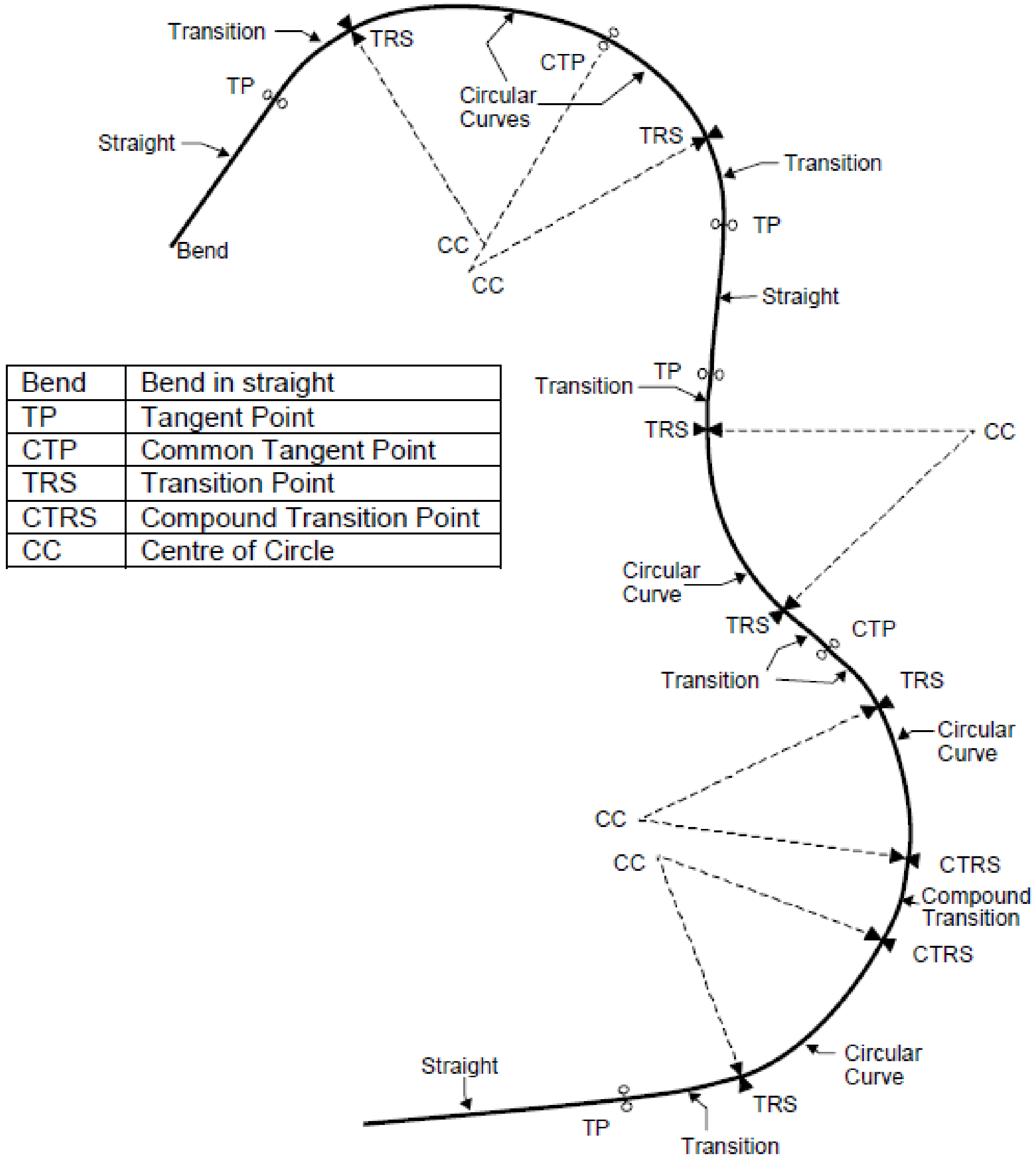
2.1 First Thoughts

The first thoughts began by considering the question of alignments, because considering the cadastral case seemed more difficult and perhaps less productive. The thoughts commenced with how to programmatically go about recovering the original intent of the alignments.

The thoughts were initially based around re-establishing the most probable location of the arcs by calculating the average radius from the given coordinates (which then provides arc length and arc angle based on these same coordinates), then determining the most probable location of the three frame points holding the calculated radius and arc length. The next step was to calculate the various parts of a transition which was tangential to the arc, with a length calculated from the radius and the chord distance (based on the given coordinates). The final step was to fill in the straight. The problem with all of this was that the transitions may not be tangential to the straights as this property was not considered when calculating the transition.

Figure 1 shows a schematic type arrangement of railway geometric segments (not to scale). Typical sizes are radius minimum 300 m, transition length maximum 100 m and segment lengths minimum 20 m. In most cases, the radius of the curve is the largest dimension, although in country locations the straight can be the largest dimension. The relocation of the intersection point can be difficult as this is often outside the boundary and marking is disturbed or destroyed.

In Figure 1, all geometry segments are tangential to the adjoining segment, except where there is a nominated bend. The various segments are placed as required to achieve the alignment required, with the geometric properties of the segments defined by key dimensions such as segment length, radius and sometimes angle at the centre of curves.



Bend	Bend in straight
TP	Tangent Point
CTP	Common Tangent Point
TRS	Transition Point
CTRS	Compound Transition Point
CC	Centre of Circle

Figure 1: Schematic railway alignment (Transport for NSW, 2019).

In the case of NSW railway transitions, the cubic parabola is used, although in road alignments it is common to use the clothoid (also known as Cornu spiral or Euler spiral, i.e. a curve that is characterised by its curvature being proportional to its length), which is also used in some other railway systems. Figure 2 shows the geometric properties for the cubic parabola, and similar properties can be derived for the clothoid. Note that there are a large number of transition types used around the world. It is expected that the methods described here can be used in all such cases, once similar properties are determined.

With reference to Figure 2, it should be noted that in the case of a cubic parabola, some terms are series expansions and as such the precision of the results is dependent on the number of terms in the equation, meaning that the final answer is similarly limited. This does not make a significant difference as there are sufficient terms to remove any practical effect, but the limitation is still there.

Parameters: Radius (R); Transition Identifier (x_c)

$$\Phi = \sin^{-1}\left[\frac{2}{\sqrt{3}}\cos\left\{\frac{\cos^{-1}\left(-\frac{3}{4}\sqrt{3}\times\frac{x_c}{R}\right)}{3}+240\right\}\right]$$

$$m = \left(\frac{\tan\Phi}{3x_c^{2}}\right)$$

$$y_c = m x_c^{3}$$

$$x^{1} = R.\sin\Phi$$

$$h = y_c + R(\cos\Phi - 1)$$

$$\theta = \tan^{-1}\left(\frac{\tan\Phi}{3}\right)$$

$$L = x_c + \frac{9}{10}m^2 x_c^{5} - \frac{9}{8}m^4 x_c^{9} + \frac{729}{208}m^6 x_c^{13} - \frac{32805}{2176}m^8 x_c^{17} + \ldots\ldots$$

$$R = \frac{\left(1+9m^2 x_c^{4}\right)^{3/2}}{6mx_c}$$

$$R_1 = \frac{\left(1+9m^2 x_1^{4}\right)^{3/2}}{6mx_1}$$

$$x_1 = L_1 - 0.9m^2L_1^{5} + 5.175m^4L_1^{9} - 43.1948m^6L_1^{13} + 426.0564m^8L_1^{17}$$

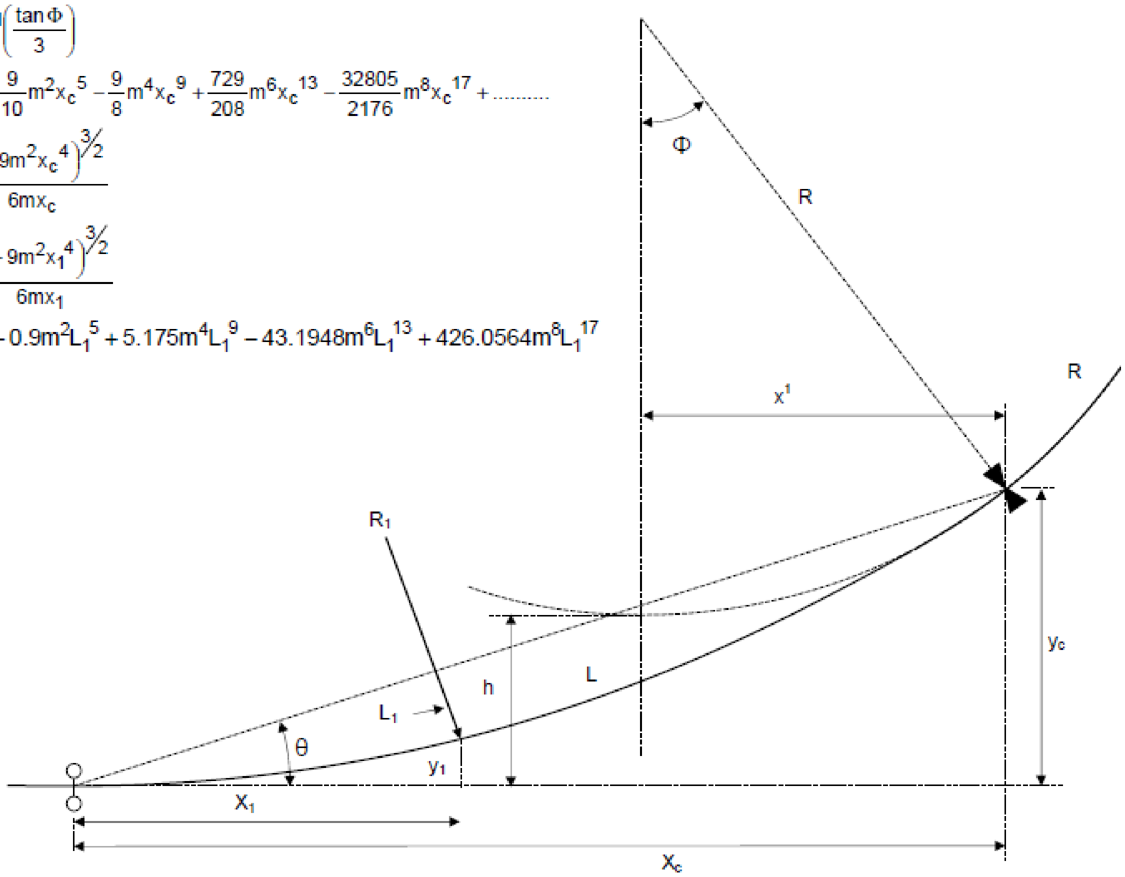


Figure 2: Cubic parabola transition (Transport for NSW, 2019).

2.2 Second Thoughts

The second thoughts continued with the view that an alternative approach was needed. The author tried to start defining the longest straights. This led to the idea of re-establishing the intersection points of the straights (as shown in Figure 3) using a combination of curve and transition properties to calculate the straights, then fill in the curve and transition geometry that fitted. While examining this process, many cases were identified where it would not deliver the desired outcome. This method also suffered from the limitation mentioned above, where the intersection points were even less often documented, and marking was often unusable. There had to be a better way.

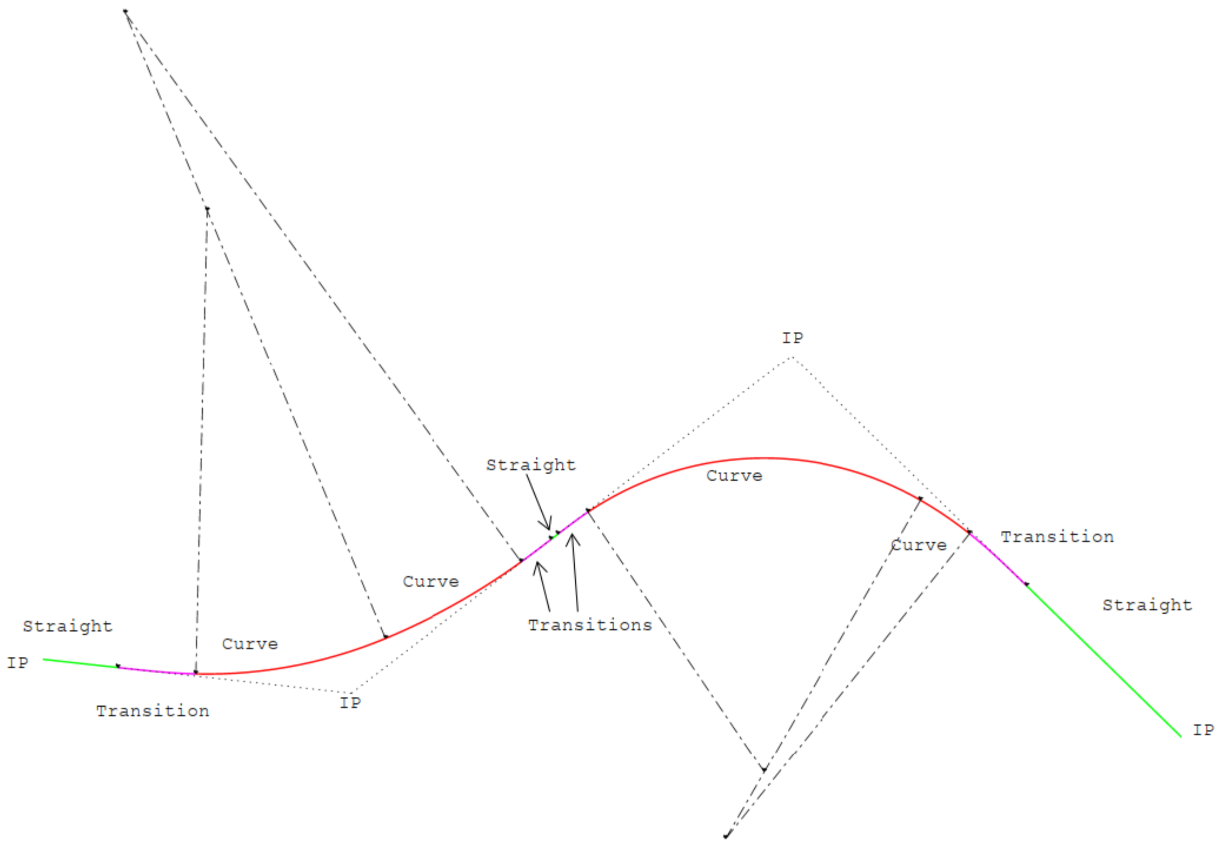


Figure 3: Alignment with intersection points.

2.3 Attempted Application of Einstein's Thought Experiment Methods

Following on from these ideas, the author realised that they had been thought experiments without being identified as such. Reflection on what had been read about Einstein's approach led to an understanding that the thought experiments had failed in the basics and were not going to provide any meaningful progress until the real problem was identified. This took quite some time, but slowly the concept of a least squares solution seemed to be the best way forward, without being sure of exactly what inputs were required. The most obvious inputs were the coordinates themselves, and some information regarding the type of geometry between the coordinates.

3 THE PROCESS OF DEVELOPING A SOLUTION

After sitting on this for some time, there was a realisation that a least squares approach would be able to solve the problem and provide a solution which statistically would be closest to the original data. This type of approach has been used by many utility organisations following Digital Cadastral Database (DCDB) updates. The utility organisations have positioned the services in their Geographic Information System (GIS) based on measurements relative to land boundaries (or occupations). With the adjustment of the spatial coordinates of the land boundaries, the service locations have to be similarly adjusted. Due to the complexity of the process, a least squares approach is often utilised. The author is aware of several papers having been written about this approach, none of which he has been able to obtain.

Several different least squares software options were considered for this approach, and DynAdjust (Fraser et al., 2025) and HAVOC (DCS Spatial Services, 2025) were shortlisted. Although a comparison of the packages' suitability for this task was intended, HAVOC was the only one that was able to be installed at the time. A comparison between these two software packages was planned, but at the time of writing this has not been possible.

Initial thoughts were to try and process cadastral data. Angles, directions and distances were straight forward, but other constraints like boundaries being parallel or perpendicular required more thought. The perpendicular case is solved by using an angle (or direction set) with a high weight assigned to these. The parallel case was more difficult with a few possibilities considered, but as of yet none have been implemented.

For the alignment case, again there are many direction sets and distances that can be implemented. However, when it came to the transition case, it was less clear which 'observations' should be used. Most road alignments utilise a clothoid as the transition, and in NSW most railway transitions are cubic parabolas. As several sample railway alignments were available, these were used for this investigation, although very similar derivations and the same methods can be used in the case of road alignments.

The approach investigated was to utilise available least squares adjustment software to adjust the initial coordinates of the frame points of the alignment, including constraints like tangential segments, to derive coordinates that were within 1 digit in the sixth decimal place of the metre (0.000 001 m). This target replicates the calculation precision in Transport for NSW (2019).

To begin the testing, simple geometries were considered without the tangential constraint. Sample data was prepared for a triangle with precise coordinates known and rounded coordinates extracted for the process. A combination of coordinates, directions and distances were prepared for input. Each of the inputs was weighted, with the aim of reproducing the known precise coordinates.

3.1 Difficulties Encountered When Using Survey Packages For This Exercise

Survey packages for least squares adjustment of observations have typically developed in such a way that makes incorporating the various uncertainties present in survey observations relatively easy. Some of the methods within such software include centring uncertainties and limitations on output precision. These properties of such software do not meet the requirements of this application and need to be manipulated to be of use. This process is not about adjusting survey observations, but rather adjusting other data (including the coordinates) to reproduce the original information.

The first test case was a simple triangle. The triangle chosen was close to an equilateral triangle to give the best chance of finding a solution (Figure 4).

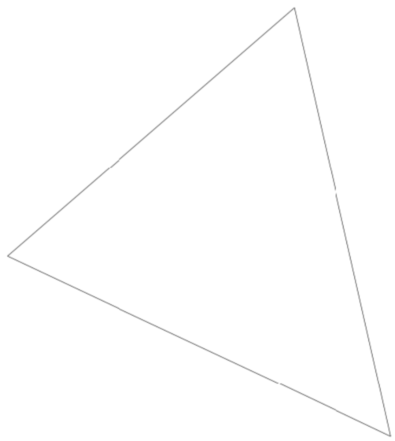


Figure 4: First triangle.

The triangle was created in 2D coordinate space. This triangle was designed to be of a size that would be similar to some of the geometries the process was to be used on. This triangle was created with coordinates that were random, although the geometry was like an equilateral triangle. The coordinates of the vertices were entered into a text file. Additional observations were needed, so the side lengths were entered (rounded to a single millimetre as this was consistent with a survey plan or drawing), and then the measured angles were entered (to 1 decimal place of a second).

All coordinates were weighted to 0.5 mm as this was the maximum expected variation of the coordinates that was desired. When assigning weights to the angles, the intention was that the angles were to be nearly fixed, as this is the sort of weighting required in the final solution, where the geometry segments would all be tangential, perpendicular or parallel.

HAVOC version 3.6 was run to adjust the data, and after several attempts a reasonable output was obtained. The resulting coordinates were closer to the original values than the rounded coordinates that were used as input. The difficulties encountered with HAVOC adjustment runs were either having the formatting and options correct or having variations in the relative weights of the inputs correct.

This identified the problem with the choice of software mentioned earlier. HAVOC was written to adjust survey data for control surveys, where results to a millimetre were the tightest expected. An option switch was added to display to 4 decimal places which helped. Another property of normal survey adjustments is entering appropriate values for the centring of instruments. This variable was not appropriate to the process being developed, and therefore centring was set to the tightest parameter. The reason for this was that the process was not about adjusting survey information, but rather trying to recreate the original geometry from one where the values were all rounded to a millimetre.

To further test this method, a copy of the data file was created with the coordinates and distances entered using centimetres instead of metres as the unit. This allowed the results to be displayed to an additional two decimal places, with the weighting changed to give a higher weight to the angles. The results showed improvement, although not by as much as was hoped. The likely reason for this is due to the small sample size and therefore insufficient data to obtain the level of results hoped for. A final adjustment at decimetre-level provided the closest results, which are shown in Table 1. These results were still less than hoped for.

Table 1: Original and adjusted coordinates for a triangle calculated at dm-level, then converted back to metres.

Original Coordinates (m)		Adjusted Coordinates (m)		Differences (m)	
E	N	E	N	E	N
365254.160440	532489.834990	365254.160140	532489.835080	0.000300	-0.000090
365282.927923	532514.745170	365282.927840	532514.745120	0.000083	0.000050
365292.609457	532471.748938	365292.609020	532471.748800	0.000437	0.000138

At this stage, several alternative uses for this method came to mind, including the re-establishment of cadastral boundaries before the application (and perhaps measurement) of field data, where plan dimensions were rounded to different levels depending on the length of the boundary and bearings were similarly rounded. This situation poses additional constraints as boundaries can be intended to be parallel or perpendicular, although shown with rounded values based on the boundary length. The required data inputs to hold these constraints have not yet been established and may have to wait for alternative least squares software written specifically for this type of task.

In Figure 5, the building parcels on the left might have parallel sides, but either or both the front and rear boundaries might be square to the side boundaries. The front and rear boundaries have the added constraint that they are straight. The residue parcel may have its longer boundary lengths rounded, and the precision of the longer boundary bearings might be tighter than the similar values for the shorter boundaries.

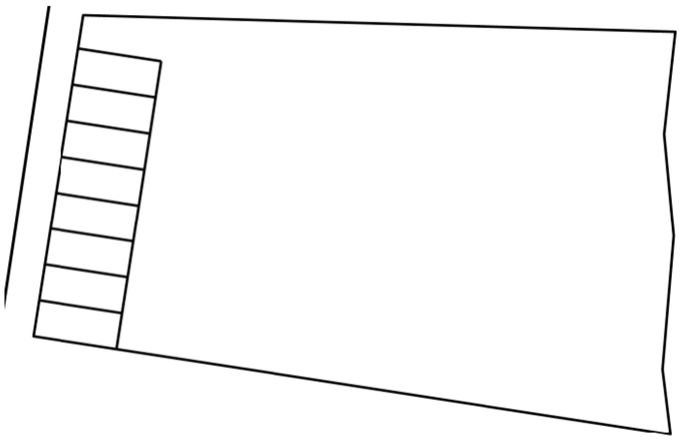


Figure 5: Typical cadastral layout where rounded dimensions could be encountered.

At this stage, the additional cadastral constraints posed a significant obstacle. Attention was therefore moved to the railway alignment cases in the hope that cadastral cases could be developed after dealing with railway alignments.

A section of a railway alignment, which was available to 6 decimal places, was chosen and the data prepared in the units of centimetres, truncating the values to fit the field sizes. Derived properties from Figure 2, together with some simpler ones for circular curves, were used to constrain the adjustment within the limitations of the survey adjustment process as described above. These produced several angles, which were entered as direction sets, and distances with additional points located in the transitions to make up the frame of the geometry.

Again, the results were encouraging but not as good as was hoped for. Tweaking the relative weighting of coordinates, angles and distances was undertaken to achieve the results that were closer to the sought-after balance of adjustments to each group of data (directions, distances and weighted coordinates).

The results achieved using this method were within approximately 0.3 mm of the original non-rounded values. This level of refinement allowed a tangential alignment to be entered into design software, although the transition lengths varied by up to 5 mm. The solution achieved was getting closer to the outcome hoped for at the beginning and provided outputs that were almost unchanged in the sense of drawing documentation. However, there were a few cases where the results led to small but noticeable differences, such as the transition lengths mentioned above. It is hoped that with further refinement of the weightings and the addition of any extra constraint data, the final plan records will be identical to the input data.

4 SUMMARY OF RESULTS SO FAR

The initial results for the triangle example identified limitations with the adopted method and did not provide encouraging results. This was possibly due to the simplicity of the example and the lack of constraints.

The examination of the railway alignment example provided more encouraging results but did not meet the initial aim of 1 unit in the sixth place of a metre coordinates. However, it did provide a closer approximation of the initial unrounded data and demonstrated that there was value in pursuing the process further. Additional examination of railway alignment data, including dual alignment with separation (track centres) as a constraint, should be pursued and incorporated into the method.

The examination of the constraints in the cadastral case demonstrated that purpose-built (or bespoke) software would be better suited to address the constraints that are not normally seen in survey data.

5 CONCLUDING REMARKS

This paper has investigated the problem of dealing with rounded values obtained on survey plans as the input for cadastral calculations, where every dimension is considered on its merits, including implied dimensions (which can sometimes be hidden). It considered the use of different methods to come up with solutions, including methods used by Albert Einstein (although not mastered by the author) and more familiar methods like least squares approaches. Potential applications of the methods were mentioned, although not all have been developed.

The process has partially delivered the expected results, with some benefits already achieved. There is the opportunity for more improvements in the process with the adjustment and balance of weighting, and the inclusion of additional geometric constraints yet to be modelled.

This investigation should be taken as a ‘call to arms’ in two ways:

- Firstly, the initial work undertaken should be continued and published to aid surveyors (especially public authority surveyors) in carrying out their functions in a timely and efficient manner.
- Secondly, all those present should be encouraged to look ‘outside the box’ and see how their training can be applied in the modern world.

ACKNOWLEDGEMENTS

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