

Performance of AUSGeoid09 in Mountainous Regions

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ABSTRACT

The Australian Height Datum (AHD) is our current national vertical datum, and AUSGeoid09 is the current quasigeoid model used to compute (normal-orthometric) AHD heights from Global Navigation Satellite System (GNSS) derived ellipsoidal heights given in the Geocentric Datum of Australia 1994 (GDA94). While previous studies have evaluated the AUSGeoid09 model across Australia, these have generally not focused on mountainous regions in particular. This paper investigates the performance of AUSGeoid09 in the Mid Hunter and Snowy Mountains regions of New South Wales, from a user's perspective. Comparisons are undertaken in an absolute sense (i.e. single point) and relative sense (i.e. height difference between two points) between AUSGeoid09-derived heights and official AHD heights on public record. The performance of AUSGeoid09 is evaluated relative to its predecessor AUSGeoid98. In both study areas, an overall improvement is evident when applying AUSGeoid09 to compute AHD heights. However, a slope was detected for AUSGeoid09 residuals in the Snowy Mountains, and it appears that the geometric component may have overcompensated for sea surface topography in this area. AUSGeoid09 generally provided AHD height differences at the ± 0.05 m to ± 0.09 m level (1 sigma) and substantially increased the percentage of GNSS-derived height differences meeting third-order differential levelling specifications. This is a very encouraging result, considering the difficulties of spirit levelling in mountainous terrain and the increasing popularity of GNSS-based height transfer in practice. However, it has been shown that some discrepancies still remain between AUSGeoid09-derived heights and AHD. It is anticipated that the new AUSGeoid2020 (to be used in conjunction with GDA2020 ellipsoidal heights) will substantially improve access to AHD via GNSS techniques. The methodology presented in this paper will be very beneficial in regards to future testing of AUSGeoid2020.

KEYWORDS: AUSGeoid09, AHD, geoid model, N values, GNSS, national datums.

1 INTRODUCTION

The Geocentric Datum of Australia 1994 (GDA94) has been our national datum since its adoption in 2000, providing fundamental geodetic infrastructure for Australia (ICSM, 2014a).

It should be noted that a new, much improved Australian national datum, GDA2020, will be released this year and is planned to be adopted by 1 January 2020 (e.g. ICSM, 2016; Gowans, 2017; Janssen, 2017).

The Australian Height Datum (AHD) is our first and only national vertical datum. It was defined by setting to zero the average Mean Sea Level (MSL) values of 32 tide gauges around Australia for a period of about two years that began in 1966 and adjusting 97,230 km of 2-way spirit levelling (Roelse et al., 1975). Now, 50 years later, it is well known that shortcomings in the AHD realisation (AHD71 for mainland Australia and AHD83 for Tasmania) resulted in MSL not being coincident with the geoid at the tide gauges involved.

These shortcomings included not considering dynamic ocean effects (e.g. winds, currents, atmospheric pressure, temperature and salinity), a lack of long-term tide gauge data, and the omission of observed gravity. This has introduced considerable distortions of up to about 1.5 m into AHD across Australia, which is therefore considered a third-order datum (e.g. Morgan, 1992; Featherstone and Filmer, 2012; Watkins et al., 2017). However, AHD continues to be a practical height datum that provides a sufficient approximation of the geoid for many surveying and engineering applications. Consequently, in practice, AHD heights are often accepted as being equivalent to orthometric heights. For a review of coordinate systems, datums and associated transformations in the Australian context the reader is referred to Janssen (2009a, 2009b). A review of Australian height systems and vertical datums can be found in Featherstone and Kuhn (2006).

Over the last two decades, Global Navigation Satellite System (GNSS) technology has become the primary positioning tool due to its accuracy, speed and accessibility. GNSS-based heights refer to a reference ellipsoid, i.e. a purely mathematical representation of the earth, and therefore have no physical meaning. In most practice, however, heights are required that correctly reflect the flow of fluids, e.g. for flood modelling, drainage and pipeline design. Hence, a reliable geoid model is required to derive AHD heights from measured ellipsoidal heights.

N values (N), also known as geoid undulations or geoid-ellipsoid separations, can be used to convert GNSS-derived ellipsoidal heights (h) to AHD heights (H) and vice versa (provided N and h refer to the same ellipsoid):

$$H = h - N \quad (1)$$

It is worth noting that H is often used to indicate orthometric heights (i.e. relative to the geoid), while Reduced Level (RL) refers to AHD heights. However, for the sake of simplicity and considering that in surveying and engineering practice both are often assumed the same, we use H to indicate AHD heights in this paper.

For many years, the use of geoid models – or quasigeoid models, see e.g. Vaniček et al. (2012) and Sjöberg (2013) for a discussion of the difference – has helped GNSS users to compute AHD heights from ellipsoidal heights. In the Australian context, AUSGeoid09 is the latest quasigeoid model to be used in conjunction with GDA94 ellipsoidal heights (Brown et al., 2011; Featherstone et al., 2011). While it should be noted that a new version, AUSGeoid2020, will soon be released for exclusive use with GDA2020 ellipsoidal heights (e.g. ICSM, 2016; Gowans, 2017; Janssen, 2017), this paper focuses on AUSGeoid09.

The performance of AUSGeoid09, along with the improvements it provides over its predecessor AUSGeoid98, has been investigated previously (e.g. Janssen and Watson, 2010, 2011; Brown et al., 2011). However, very few studies have focused on mountainous regions. Considering that gravity can change dramatically within a few kilometres on the earth's surface in Australia (Darbeheshti and Featherstone, 2009), especially in mountainous terrain, and that observed gravity data are generally sparse in these areas, it is necessary to evaluate the performance of AUSGeoid09 in mountainous regions in particular.

Geoid or quasigeoid models are commonly verified by using GNSS and orthometric height data. This can be done in an absolute and relative sense (Featherstone, 2001): An absolute verification estimates the accuracy and precision of the (quasi)geoid, with respect to the geocentric ellipsoid, using GNSS networks that have been tied to an (inter)national reference frame and spirit-levelled orthometric heights that have been tied to the (national) vertical datum. A relative verification utilises GNSS-derived ellipsoidal height differences and spirit-levelled orthometric height differences to estimate the accuracy and precision of the (quasi)geoid gradients.

This paper evaluates the performance of AUSGeoid09 in the Mid Hunter and Snowy Mountains regions of New South Wales (NSW), from a user's perspective. Comparisons are undertaken in an absolute sense (i.e. single point) and relative sense (i.e. height difference between two points) between AUSGeoid09-derived heights and official AHD heights on public record in the Survey Control Information Management System (SCIMS – see Kinlyside, 2013). The performance of AUSGeoid09 is evaluated relative to its predecessor AUSGeoid98. It should be noted that this paper combines the main findings of the absolute and relative investigations previously reported in two separate journal papers (Sussanna et al., 2014, 2016).

2 AUSGeoid09

AUSGeoid09 was released in March 2011 by Geoscience Australia, replacing the previous model AUSGeoid98 (Featherstone and Guo, 2001). Both models refer to the GRS80 ellipsoid, which was adopted as the reference ellipsoid for GDA94, and cover the same geographical area between 108°E and 160°E longitude and between 8°S and 46°S latitude. However, AUSGeoid09 is provided as a 1' by 1' grid (approximately 1.8 by 1.8 km), making it four times denser than its predecessor (Featherstone et al., 2011).

Previous versions of AUSGeoid were predominantly gravimetric-only quasigeoids, and it was assumed that these were sufficiently close approximations of AHD – an assumption we now know to be incorrect because AHD is not as closely aligned to the geoid as we would like. In contrast, AUSGeoid09 is a combined gravimetric-geometric quasigeoid, providing a direct connection to AHD and thereby allowing a more reliable determination of AHD heights from GNSS observations (Brown et al., 2011). The empirically derived geometric component accounts for the offset between the gravimetric quasigeoid and AHD, which is predominantly caused by AHD not taking into account sea surface topography including the differential heating of the oceans.

Since the warmer or less dense water off northern Australia is about 1 metre higher than the cooler or denser water off southern Australia, AHD is about 0.5 m above the quasigeoid in northern Australia and roughly 0.5 m below the quasigeoid in southern Australia (Brown et

al., 2011; Janssen and Watson, 2010, 2011). The introduction of the geometric component takes care of most of this 1-metre trend across Australia (0.6-metre trend across NSW), thereby providing a better overall fit to AHD.

AUSGeoid09 has been shown to convert ellipsoidal heights to AHD heights with an accuracy of ± 0.03 m (1 sigma), or ± 0.06 m at the 95% confidence interval (CI), with the exception of some pockets where the misfit can be larger than ± 0.1 m (1 sigma) or ± 0.2 m (95% CI) due to errors caused by factors such as the ageing levelling network, geoid height variability or data deficiency (Brown et al., 2011). Using a more practical approach, Janssen and Watson (2010, 2011) found that AUSGeoid09 generally allows GNSS-based height determination in NSW at the ± 0.05 m level (1 sigma) or ± 0.10 m (95% CI). In contrast, its predecessor AUSGeoid98 only provides an absolute accuracy of ± 0.4 m (Featherstone and Guo, 2001; Featherstone et al., 2001).

Using Network Real Time Kinematic (NRTK) GNSS observations in the Blue Mountains area of NSW, Allerton et al. (2015) found that AUSGeoid09 allows AHD height determination at the ± 0.03 m level (1 sigma) or ± 0.06 m (95% CI) in flat terrain and at the ± 0.06 m level (1 sigma) or ± 0.12 m (95% CI) in mountainous terrain. However, results also indicate that there is room for improvement in regards to future versions of the AUSGeoid model for elevations above 500 m. In this paper, we use a much larger dataset to investigate AUSGeoid09 performance in mountainous areas.

3 ABSOLUTE AND RELATIVE GNSS HEIGHTING

In an absolute sense, also known as the single point approach, the GNSS-derived ellipsoidal height at a point can be converted to a (normal-orthometric) AHD height using equation (1). This is generally applied when users connect to a GNSS Continuously Operating Reference Station (CORS) network such as CORSnet-NSW (Janssen et al., 2016; DFSI Spatial Services, 2017), e.g. via NRTK observations, or use Precise Point Positioning (PPP) to derive AHD heights.

In a relative sense, the AHD height of a point B is calculated relative to another point A with known AHD height, using the differences (Δ) in GNSS-derived ellipsoidal heights and N values supplied by the geoid model:

$$H_B = H_A + \Delta H_{AB} \quad \text{with} \quad \Delta H_{AB} = \Delta h_{AB} - \Delta N_{AB} \quad (2)$$

The well-known advantage of the relative method is that simultaneous observations at both points minimise most of the systematic errors by virtue of the difference, as is the case with GNSS baseline processing. The absolute N values at both points may have relatively large errors, but the height of point B is only contaminated by the small difference of these errors (ignoring any GNSS observational errors). Figure 1 illustrates these two approaches.

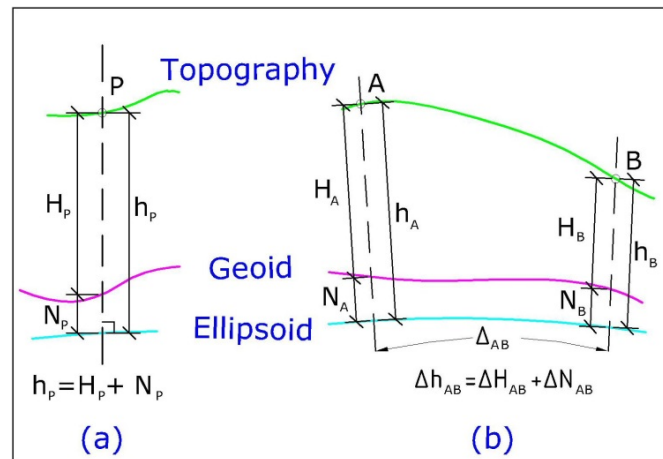


Figure 1: Relationship between ellipsoidal height (h), normal-orthometric AHD height (H) and geoid-ellipsoid separation (N) for (a) absolute and (b) relative GNSS heighting.

4 METHODOLOGY: ABSOLUTE AND RELATIVE GEOID MODEL VERIFICATION

4.1 Absolute Verification

Due to the increased use of CORS networks, the absolute accuracy of N values is now more important than ever for GNSS-based AHD height determination (Janssen and Watson, 2010, 2011). In an absolute sense, AUSGeoid09 performance was verified based on the comparison of a network of GNSS observations and official AHD heights published in SCIMS. Using equation (1), AHD-derived N values (N_{AHD}) were computed by subtracting the published AHD height (H_{AHD}) at each checkpoint from the ellipsoidal height (h) obtained from a least squares network adjustment using the GeoLab software (BitWise Ideas, 2017).

These N_{AHD} values were compared with N values computed using AUSGeoid09 (and AUSGeoid98) to determine residuals ($N_{AG} - N_{AHD}$) for the following two tests, both statistically and graphically (Featherstone and Guo, 2001):

- Comparison for all checkpoints.
- Comparison as a function of AHD height.

Descriptive statistics were computed to obtain a numerical representation of the population sample, as previously adopted by Brown et al. (2011). Z-statistics were employed to identify any outliers, in this paper defined as three times larger than the standard deviation. Since it is necessary to consider residuals of different signs, the Root Mean Square (RMS) was also utilised.

Generally speaking, the primary aim of the absolute verification was to quantify the accuracy of AUSGeoid09 in regards to computing AHD heights at single points in mountainous regions. Furthermore, a comparison between AUSGeoid09 and its predecessor AUSGeoid98 was performed to quantify the expected improvement in mountainous areas.

4.2 Relative Verification

In spite of the growing popularity of CORS networks and PPP for GNSS-based height transfer, surveyors often continue to use the relative GNSS heighting method to ‘carry’ heights from established marks to unestablished marks via baseline processing. As indicated

earlier, this concept is less affected by errors in the N values since common systematic errors are minimised by virtue of differencing.

GNSS baselines were observed between points to obtain the differences in ellipsoidal heights, which were then converted into AHD height differences using the computed N values. This can be applied over all possible mark-to-mark vector combinations within a network of control points. However, a least squares adjustment is required to create a network with consistent ellipsoidal heights. The number of possible vector combinations is $n(n-1)/2$ where n denotes the number of points.

AUSGeoid09 and AUSGeoid98 performance was investigated by comparing between any two points the difference in GNSS-derived AHD heights (ΔH_{GNSS}) computed using equation (2) with the difference in official AHD heights (ΔH_{AHD}) for these two points. Residuals (R) were then computed as follows:

$$R = \Delta H_{AB(GNSS)} - \Delta H_{AB(AHD)} \quad (3)$$

The following four tests were performed, both statistically and graphically, and analysed based on descriptive statistics:

- Comparison for all observed baselines.
- Comparison for all possible mark-to-mark combinations.
- Comparison for all possible mark-to-mark combinations up to 100 km in length.
- Comparison for all possible mark-to-mark combinations as a function of AHD height difference.

The primary aim of the relative verification was to quantify the accuracy of AUSGeoid09 in regards to computing AHD height differences in mountainous terrain. A comparison between AUSGeoid09 and its predecessor AUSGeoid98 was performed to quantify the expected improvement in mountainous areas. Furthermore, the baseline residuals were compared to the allowable misclose of third-order differential levelling (ICSM, 2007), i.e. the maximum allowable misclose of $12\sqrt{d}$ (d = distance in km, here calculated on the GRS80 ellipsoid using Vincenty's inverse formula). Residuals were also expressed as parts per million (ppm).

5 STUDY AREAS AND DATASETS

The performance of AUSGeoid09 in mountainous regions was evaluated in two study areas located in NSW (Figure 2). Both study areas represent typical mountainous terrain conditions encountered in Australia and exhibit large differences in elevation. Spatial Services, a unit of the NSW Department of Finance, Services and Innovation (DFSI), provided two GNSS network datasets collected over many years, together encompassing 186 survey marks with known AHD heights of sufficient quality (Class C Order 3 or better) on public record in SCIMS. For a discussion of the terms class and order, the reader is referred to ICSM (2007) and Dickson (2012). While it is acknowledged that ICSM (2007) has recently been superseded by ICSM (2014b), this update does not affect the outcome of the analysis presented here.

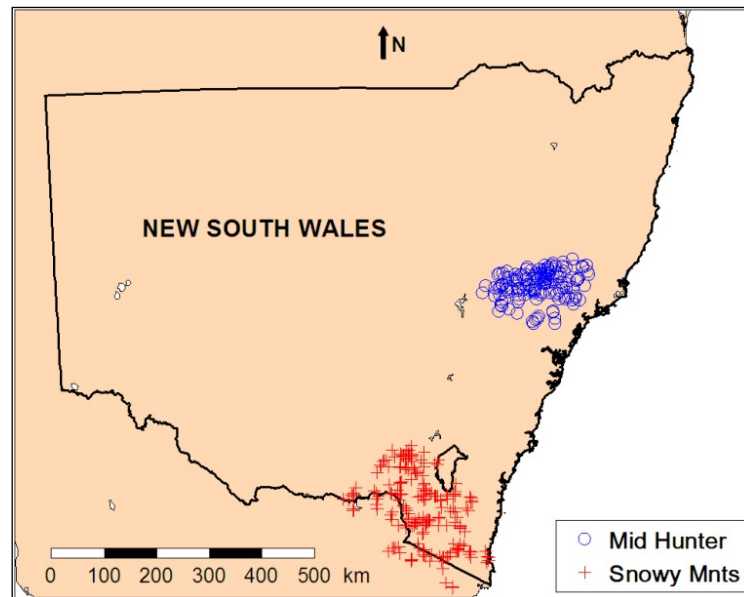


Figure 2: Location of the two study areas in NSW.

The Mid Hunter GNSS network adjustment covers an area of approximately 13,000 km², stretching from about 115 km south of the Mount Royal National Park to 170 km east of Mudgee. The terrain is mainly composed of valleys and mountains with elevations ranging between 20 m and 1,400 m. This dataset consists of 327 independent GNSS baselines observed between 147 marks. Of these, 82 SCIMS marks have known AHD heights (C3 or better), including 40 spirit-levelled marks of classification LCL3 or better.

The Snowy Mountains GNSS network adjustment covers an area of about 35,000 km², approximately bounded in the north by Tumut and the ACT border to Cooma, and in the south by Albury and the Victorian border towards the coast. The terrain exhibits an undulated topography composed of mountains reaching a peak of 2,228 m (Mt Kosciuszko) and low valleys with elevations of about 200 m. The GNSS dataset consists of 629 independent baselines observed between 263 marks. Of these, 104 SCIMS marks have known AHD heights (C3 or better), including 94 spirit-levelled marks of classification LCL3 or better.

In total, across both study areas, this provided 186 checkpoints with known AHD heights of sufficient quality for a practical AUSGeoid09 performance verification and a comparison to AUSGeoid98. It should be noted that while some of the GNSS data used in this study contributed to the generation of AUSGeoid09, the datasets are considered sufficiently independent for the purpose of this study.

6 DATA PROCESSING AND ANALYSIS

The two GNSS networks used in this study were subject to several adjustments performed using the GeoLab least squares adjustment software (BitWise Ideas, 2017). These adjustments were constrained to the national datum (GDA94) via several AUSPOS solutions (GA, 2017), i.e. 7 and 27 marks in the Mid Hunter and Snowy Mountains networks respectively. N values were computed based on bi-cubic interpolation, using both AUSGeoid09 and AUSGeoid98 to enable comparison between the two models. The resulting GNSS-derived AHD heights are therefore independent of the official AHD heights on public record.

Both the Mid Hunter and the Snowy Mountains networks generated Class A surveys as per ICSM (2007). The ellipsoidal heights for the 104 checkpoints in the Snowy Mountains network displayed an average uncertainty of ± 0.016 m (1 sigma) or ± 0.031 m (95% CI). The Mid Hunter network performed slightly better, resulting in an average uncertainty of ± 0.012 m (1 sigma) or ± 0.024 m (95% CI) for the 82 checkpoints.

6.1 Absolute Verification

For absolute verification, N_{AHD} values were computed at each checkpoint from the ellipsoidal heights obtained via the constrained least squares adjustment and published AHD heights at these survey marks using equation (1). On the same checkpoints, N values (N_{AG}) were also obtained from AUSGeoid09 and AUSGeoid98, and then compared to their N_{AHD} values.

6.1.1 Absolute Comparison for all Checkpoints

In order to examine the distribution of the residuals ($N_{AG} - N_{AHD}$), a Kurtosis test was performed. A negative Kurtosis value indicates flatness with a large number of residuals concentrated along the side of a normal distribution, while a positive value denotes a sample of a peak with the majority of the residuals concentrated in the proximity of the mean (DeCarlo, 1997). It was found that the AUSGeoid09 residuals in both study areas are more consistent with a normal distribution with a large amount of residuals close to the mean, while AUSGeoid98 residuals denoted flatness with a large amount of residuals along the side of the normal distribution.

The comparison of the two quasigeoid models in the Mid Hunter study area shows a substantial improvement using AUSGeoid09 over its predecessor, evidenced by the standard deviation dropping from ± 0.074 m to ± 0.040 m. The RMS for AUSGeoid09 indicates an improvement factor of 6 compared to AUSGeoid98. Neither of the two models indicated any outliers of three times their standard deviation.

As evident from Figure 3, the AUSGeoid98 residuals are heavily positive at around 0.25 m, while the AUSGeoid09 residuals are well distributed around a near-zero mean. AUSGeoid09 provides an exceptional improvement over AUSGeoid98, showing no trends and exhibiting consistent residuals across the study area.

Figure 3a suggests the existence of a slope in AUSGeoid98 residuals in the longitudinal direction, dipping towards the east of the network, while Figure 3b indicates a rise in the middle of the study area's latitudinal extent. The heavily positive AUSGeoid98 residuals are consistent with the geometric component of AUSGeoid09 generally amounting to about -0.2 m in this area (Brown et al., 2011), clearly showing the beneficial effect the introduction of the geometric component has on GNSS-derived AHD height determination in this area.

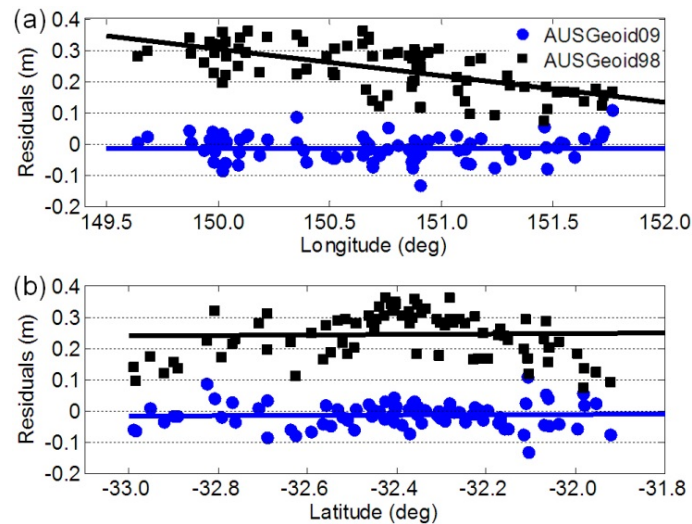


Figure 3: Mid Hunter verification: Absolute residuals between quasigeoid-derived (AUSGeoid09 and AUSGeoid98) N values and N_{AHD} values for (a) increasing longitudes and (b) decreasing southern latitudes.

The comparison of the two quasigeoid models in the Snowy Mountains study area reveals a moderate improvement of AUSGeoid09 over AUSGeoid98 with the standard deviation dropping from ± 0.090 m to ± 0.070 m. However, no improvement was detected in the RMS. While neither model showed any outliers greater than three times the standard deviations, the mean of the residuals is closer to zero for AUSGeoid98 than AUSGeoid09 because the AUSGeoid98 residuals are almost equally balanced between positive and negative values ranging between -0.224 m and 0.165 m. In contrast, the majority of AUSGeoid09 residuals are negative ranging from -0.284 m to 0.059 m.

Figure 4a illustrates the residuals of both quasigeoid models as a function of their longitudinal position. It is evident that AUSGeoid98 residuals show a relatively large scatter but no trend. On the other hand, AUSGeoid09 residuals exhibit a lesser spread but show a trend where the residuals seem to increase (larger negative values) in the eastern direction.

Both models show a similar trend as a function of the latitudinal position, with residuals decreasing (smaller negative values) in the northern direction (Figure 4b). A closer evaluation of the residuals identifies that AUSGeoid98 residuals are of larger magnitude and larger spread from the mean, while AUSGeoid09 residuals are more closely aligned with the mean, the only exception being two checkpoints at latitude -37.0° (and longitude 149.9°). If these two residuals are removed, the range in AUSGeoid09 residuals decreases by about 75 mm and the standard deviation improves to ± 0.065 m. However, there is not enough evidence supporting a gross error within these two checkpoints to justify their removal.

Figure 4 shows that a slope is evident within AUSGeoid09 residuals from the north-west to the south-east corner, with the majority of negative residuals situated at the south-east corner of the study area. This direction is perpendicular to the known general south-west to north-east slope present in the gravimetric component of AUSGeoid09 (and more generally the geoid across Australia). While this slope indicates that a small residual geometric effect may be present in this area, the sample is not large enough to identify any anomalies in AUSGeoid09 with any certainty. The small offset between the two models is consistent with the geometric component of AUSGeoid09, generally amounting to about -0.1 m or less in this area (Brown et al., 2011). This suggests that the geometric component of AUSGeoid09 may have overcompensated for the effect of sea surface topography in this case.

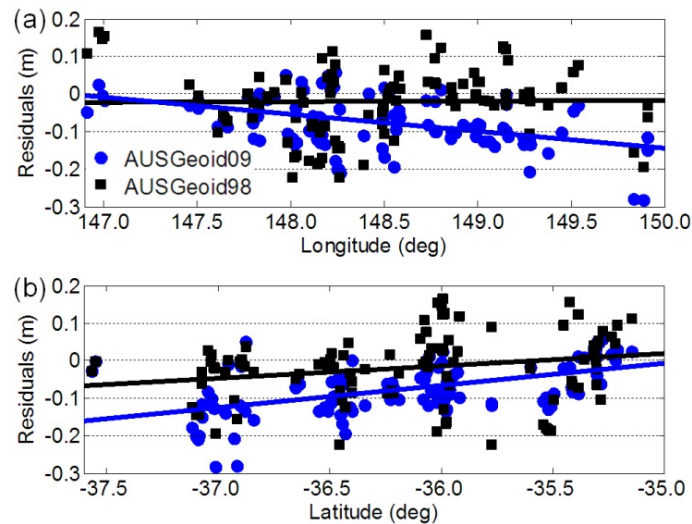


Figure 4: Snowy Mountains verification: Absolute residuals between quasigeoid-derived (AUSGeoid09 and AUSGeoid98) N values and N_{AHD} values for (a) increasing longitudes and (b) decreasing southern latitudes.

6.1.2 Absolute Comparison as a Function of AHD Height

In mountainous regions, it is useful to investigate the performance of the two quasigeoid models as a function of AHD height. While it is recognised that the sample of checkpoints decreases considerably with increasing elevation, this will provide an indication of how well the two models fit AHD in undulating terrain. Following the approach taken by Featherstone and Guo (2001), Figure 5 illustrates the absolute N value residuals for both quasigeoid models as a function of AHD height for the 82 checkpoints located in the Mid Hunter study area. Figure 6 shows the corresponding results for the 104 checkpoints in the Snowy Mountains study area.

It is confirmed that AUSGeoid09 produces a smaller scatter in the residuals and generally provides a better fit (i.e. residuals closer to zero), especially for higher elevations. This is particularly evident in the Mid Hunter study area, clearly showing the improvement obtained when using AUSGeoid09. As previously mentioned, the offset between the two models is consistent with the magnitude of the geometric component of AUSGeoid09. This again demonstrates the benefit of introducing the geometric component, but also indicates that it may have overcompensated for the sea surface topography effect in the Snowy Mountains study area. However, it should be noted that these results are not a true representation of a continuous elevation model because the checkpoints are located across a large area and both datasets contain a small number of checkpoints at the higher elevations.

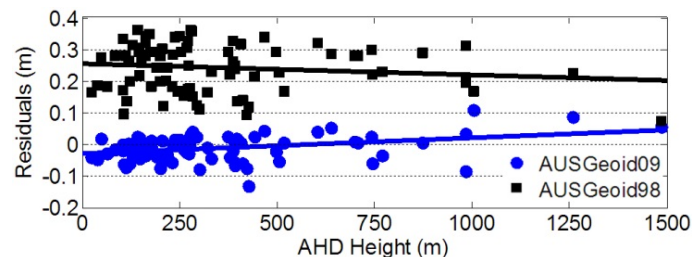


Figure 5: Mid Hunter verification: Absolute residuals between quasigeoid-derived (AUSGeoid09 and AUSGeoid98) N values and N_{AHD} values as a function of AHD height.

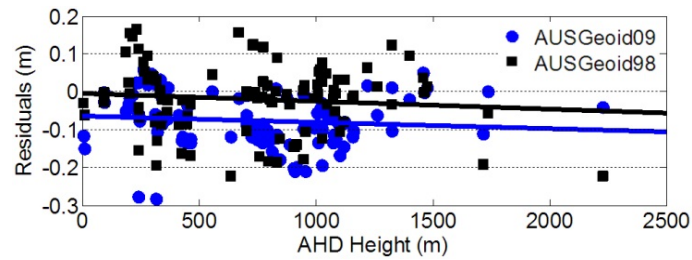


Figure 6: Snowy Mountains verification: Absolute residuals between quasigeoid-derived (AUSGeoid09 and AUSGeoid98) N values and N_{AHD} values as a function of AHD height.

These findings are supported by investigating descriptive statistics of the absolute residuals, calculated for all checkpoints in increments of 200 m in AHD height (values not shown here – for details see Sussanna et al., 2014). In the Mid Hunter study area, both quasigeoid models demonstrate relatively stable and consistent sets of statistics with increasing elevation. AUSGeoid09 shows substantial improvements in the mean, standard deviation, RMS and also the range of residuals. The large positive bias in the mean of the AUSGeoid98 residuals has been successfully accounted for by AUSGeoid09.

In the Snowy Mountains study area, AUSGeoid09 shows improvements over AUSGeoid98 in terms of standard deviation and range, particularly for higher elevations. The AUSGeoid09 statistics are also generally more stable with increasing elevation. However, the RMS only shows improvement for the highest elevations. As mentioned earlier, the mean of the AUSGeoid09 residuals is noticeably biased to the negative.

6.2 Relative Verification

Relative verification is based on GNSS baselines stemming from the constrained least squares adjustment. AUSGeoid09 and AUSGeoid98 performance was investigated by comparing between any two points the difference in GNSS-derived AHD heights (ΔH_{GNSS}) computed using equation (2) with the difference in official AHD heights (ΔH_{AHD}) for these two points. The residuals, computed using equation (3), were then compared in four different tests.

6.2.1 Relative Comparison for all Observed Baselines

In the Mid Hunter network, analysis of the 104 observed baselines shows that AUSGeoid09 and its predecessor AUSGeoid98 perform at the same level as far as recovering AHD heights from GNSS is concerned, with an RMS of about 0.046 m. AUSGeoid09-derived results included no outliers and one outlier was identified when using AUSGeoid98. On average, AUSGeoid98 actually performs slightly better when the average baseline length of 13.1 km is considered (3.7 ppm vs. 4.0 ppm). However, this 0.3 ppm difference is equivalent to less than 4 mm in height over the average baseline length, i.e. less than 10% of the allowable misclose. Both models perform similarly in meeting third-order differential levelling specifications, with about 65% of baselines within specifications. Interestingly, the highest AUSGeoid98 residuals occur at different elevation ranges without any particular pattern, while the highest AUSGeoid09 residuals occur for baselines with height differences between 400 m and 750 m. However, it should be noted that this finding is based on only 6 out of 104 observed baselines and not evident when all possible mark-to-mark combinations are investigated.

In the Snowy Mountains network, analysis of the 66 observed baselines demonstrates that AUSGeoid09 performs moderately better than its predecessor AUSGeoid98. AUSGeoid09-

derived results showed no outliers, while one outlier was identified when using AUSGeoid98. The RMS dropped from 0.064 m to 0.051 m, resulting in an improvement factor of almost 1.3. AUSGeoid09 also performs slightly better when the average baseline length of 7.5 km is considered (7.9 ppm vs. 8.1 ppm) – this 0.2 ppm difference is equivalent to only 1.5 mm in height over the average baseline length (less than 5% of the allowable misclose). AUSGeoid09 produced better results in regards to meeting third-order differential levelling specifications, with about 61% vs. 56% of baselines within the maximum allowable misclose.

6.2.2 Relative Comparison for all Possible Mark-to-Mark Combinations

In the Mid Hunter network, a total of 3,320 possible mark-to-mark vector combinations were analysed. Although AUSGeoid09-derived results identified 15 outliers (none were identified when AUSGeoid98 was applied), it is evident that AUSGeoid09 is far superior to its predecessor. The RMS dropped from 0.105 m to 0.056 m, i.e. an improvement factor of 1.9. Considering the average vector length of 74.9 km, using AUSGeoid09 improved performance by a factor of 1.7, from 1.5 ppm to 0.9 ppm. This 0.6 ppm difference translates into 45 mm in height over the average vector length, thus having a substantial effect (43% of the allowable misclose). Furthermore, 90% of the AUSGeoid09-derived residuals met third-order differential levelling specifications (Figure 7a), while only 61% of AUSGeoid98-derived residuals achieved the same (Figure 7b).

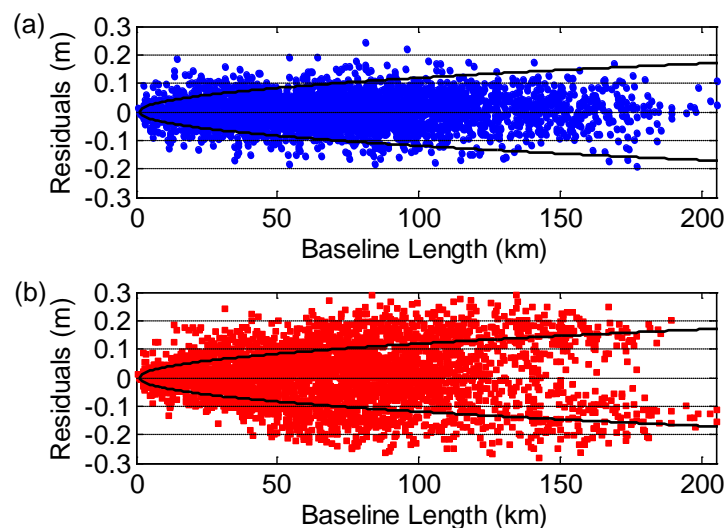


Figure 7: Mid Hunter residuals between (a) AUSGeoid09- and (b) AUSGeoid98-derived and official AHD height differences over 3,320 possible vectors. The unbroken line describes the allowable third-order differential levelling misclose.

In the Snowy Mountains network, a total of 5,356 possible mark-to-mark combinations were analysed. Again, although AUSGeoid09-derived results identified 15 outliers (3 were identified when using AUSGeoid98), it is evident that AUSGeoid09 performs much better than AUSGeoid98. The RMS dropped from 0.127 m to 0.099 m, translating into an improvement factor of 1.3. Considering the average vector length of 113.4 km, AUSGeoid09 improved performance from 1.4 ppm to 1.1 ppm. This 0.3 ppm difference is equivalent to 34 mm in height over the average vector length, i.e. 27% of the allowable misclose. Furthermore, 77% of the AUSGeoid09-derived residuals met third-order differential levelling specifications (Figure 8a), while only 64% of AUSGeoid98-derived residuals fell within these bounds (Figure 8b).

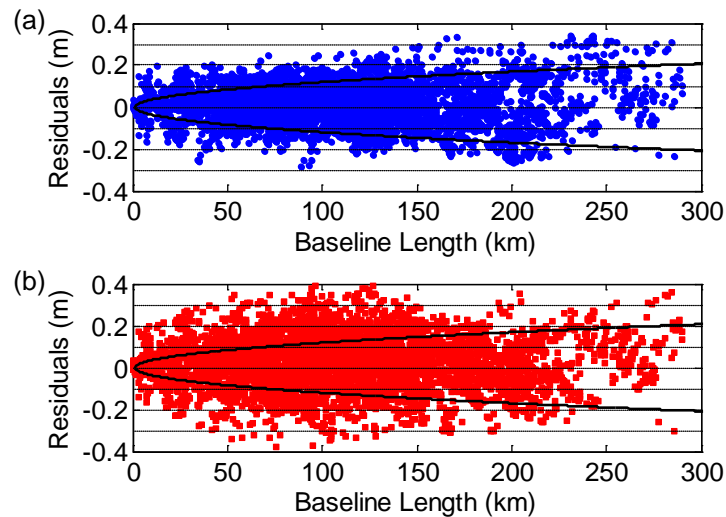


Figure 8: Snowy Mountains residuals between (a) AUSGeoid09- and (b) AUSGeoid98-derived and official AHD height differences over 5,356 possible vectors. The unbroken line describes the allowable third-order differential levelling misclose.

6.2.3 Relative Comparison for all Possible Mark-to-Mark Combinations up to 100 km

The comparison over all possible mark-to-mark combinations (section 6.2.2) includes long vectors that are often well above 100 km in length. Generally speaking, it is unlikely that GNSS users perform network adjustments with baselines of this length, unless they contribute to state-wide or national control networks. Therefore, this third test was conducted to verify the performance of AUSGeoid09 based on the subset of all possible vectors up to 100 km in length, providing a more realistic GNSS network performance evaluation approach from a practical point of view.

In the Mid Hunter network, a total of 2,526 possible vectors with lengths up to 100 km were analysed. Although AUSGeoid09-derived results identified 12 outliers (none were identified when AUSGeoid98 was applied), AUSGeoid09 again shows superior performance to AUSGeoid98. The RMS dropped from 0.095 m to 0.055 m, i.e. an improvement factor of 1.7. Considering the average vector length of 57.4 km, using AUSGeoid09 improved performance from 1.6 ppm to 1.1 ppm (improvement factor of 1.5). This 0.5 ppm difference translates into more than 28 mm in height over the average vector length, thus having a substantial effect (31% of the allowable misclose). Moreover, 87% of the AUSGeoid09-derived residuals met third-order differential levelling specifications, while only 61% of AUSGeoid98-derived residuals achieved the same.

In the Snowy Mountains network, a total of 2,361 possible vectors up to 100 km length were analysed. For both models, a similar number of outliers were identified (7 vs. 6). Again, it is evident that AUSGeoid09 performs much better than AUSGeoid98, with the RMS dropping from 0.122 m to 0.082 m (improvement factor of 1.5). Considering the average vector length of 60.9 km, AUSGeoid09 improved performance from 2.3 ppm to 1.7 ppm. This 0.6 ppm difference is equivalent to more than 36 mm in height over the average vector length, again having a substantial effect (38% of the allowable misclose). In this case, 71% of the AUSGeoid09-derived residuals met third-order differential levelling specifications, compared to only 53% of AUSGeoid98-derived residuals falling within specifications.

6.2.4 Relative Comparison for all Possible Mark-to-Mark Combinations as a Function of AHD Height Difference

Investigating the performance of the two quasigeoid models as a function of AHD height difference provides an indication of how well the two models fit AHD in undulating terrain. Again following the approach taken by Featherstone and Guo (2001), Figure 9 illustrates the residuals for both quasigeoid models as a function of AHD height difference for the 3,320 vectors located in the Mid Hunter study area. Figure 10 shows the corresponding results for the 5,356 baselines in the Snowy Mountains study area.

It is confirmed that AUSGeoid09 produces a smaller scatter or variation in the residuals and generally provides a better fit (i.e. residuals closer to zero). For higher elevation changes, this is particularly evident in the Snowy Mountains study area, clearly showing the improvement obtained when using AUSGeoid09.

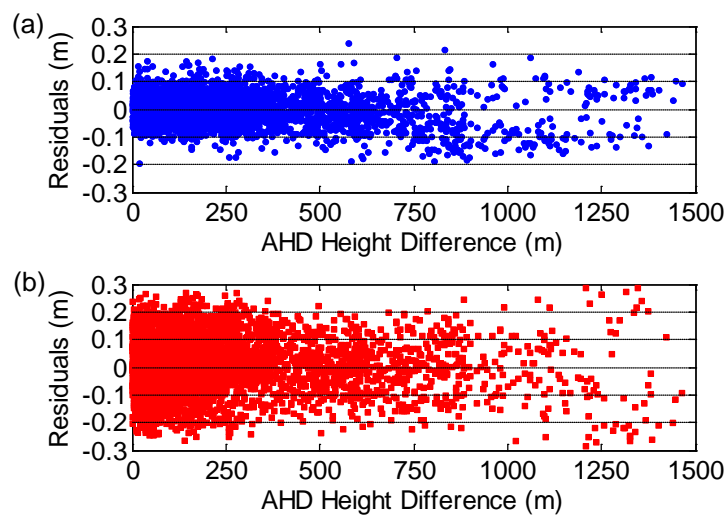


Figure 9: Mid Hunter residuals between (a) AUSGeoid09- and (b) AUSGeoid98-derived and official AHD height differences over 3,320 possible vectors as a function of AHD height difference.

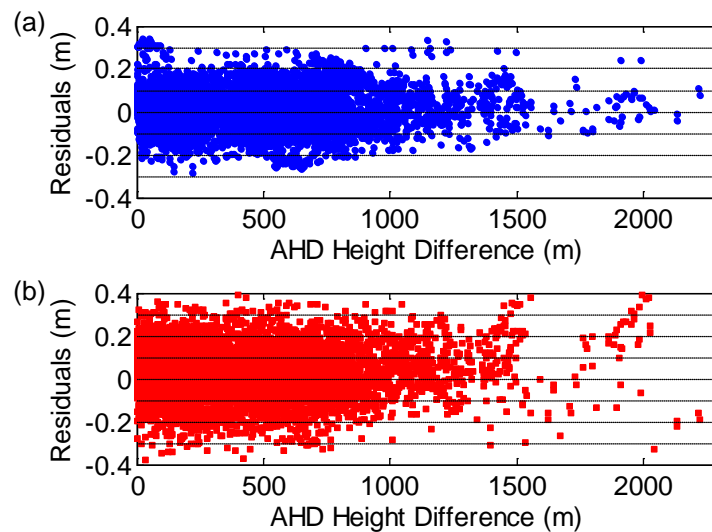


Figure 10: Snowy Mountains residuals between (a) AUSGeoid09- and (b) AUSGeoid98-derived and official AHD height differences over 5,356 possible vectors as a function of AHD height difference.

These findings are supported by investigating descriptive statistics of the residuals between GNSS-derived and official AHD height differences, calculated for all possible vectors in increments of 200 m in AHD height difference (values not shown here – for details see Sussanna et al., 2016). In the Mid Hunter network, both quasigeoid models demonstrate relatively stable and consistent sets of statistics with increasing height differences. RMS values for AUSGeoid09 slowly increase with increasing height difference, and the largest values occur for baselines with height differences of 800 m to 1,200 m. RMS values for AUSGeoid98 are larger but remain consistent until increasing considerably for elevation changes above 1,200 m. AUSGeoid09 shows substantial improvements in the standard deviation, RMS and the range of residuals across all height increments when compared to AUSGeoid98.

In the Snowy Mountains network, both quasigeoid models also present relatively stable and consistent statistics. RMS values for AUSGeoid09 show no evidence of deterioration with increasing elevation change. However, AUSGeoid98-derived results noticeably deteriorate for elevation changes above 1,400 m, indicating a much improved fit to AHD when using AUSGeoid09 for GNSS-based height transfer in heavily undulating terrain. Across all height increments, AUSGeoid09 again shows substantial improvements over AUSGeoid98 in the standard deviation, RMS and the range of residuals.

7 CONCLUDING REMARKS

By examining two extensive datasets located in NSW, this paper has investigated the performance of the AUSGeoid09 quasigeoid model in mountainous regions and compared it to its predecessor AUSGeoid98, from a user's perspective. As expected, AUSGeoid09 has demonstrated increased consistency and accuracy compared to its predecessor, owing to the inclusion of a geometric component, a larger amount of input data and its higher density.

In the absolute verification (i.e. based on absolute N values and AHD heights), this improvement was more evident in the Mid Hunter than in the Snowy Mountains. In the Mid Hunter study area, AUSGeoid09 showed a substantial improvement over AUSGeoid98 in retrieving AHD heights from GNSS-derived ellipsoidal heights, evidenced by the standard deviation dropping from ± 0.074 m to ± 0.040 m and RMS values improving by a factor of 6. No trend was evident as a function of the horizontal position of the checkpoints, and relatively stable and consistent sets of statistics were obtained for increasing elevations. AUSGeoid09 clearly demonstrated the benefit of its geometric component on GNSS-derived AHD height determination in this area.

In the Snowy Mountains study area, AUSGeoid09 provided moderate improvement over AUSGeoid98, with the standard deviation dropping from ± 0.090 m to ± 0.070 m. However, it should be noted that the majority of AUSGeoid09 residuals were negative, rather than evenly distributed around a zero mean. This suggests that the geometric component of AUSGeoid09 may have overcompensated for sea surface topography in this case. The dataset also detected a slope in AUSGeoid09 residuals from the north-west to the south-east corner, indicating that a small residual geometric effect may be present in this area. However, the sample size is not sufficiently large to identify any anomalies in AUSGeoid09 with any certainty. It is recognised that the terrain correction (TC) is likely to represent a more prominent error source for the N values in this study area. The accuracy of its computation is dependent on the integrity of the Digital Elevation Model (DEM) in the region, and for the rugged terrain of the

Snowy Mountains this TC element is expected to show a larger degree of uncertainty than in the Mid Hunter.

In this context, it is worth noting that DFSI Spatial Services is currently undertaking the Surface Model Enhancement (SME) Project, which utilises a variety of technology including aerial imagery and Light Detection and Ranging (LiDAR) to create a high-resolution, state-wide Digital Surface Model (Powell, 2017). This is expected to significantly improve the quality of the DEM across the entire State, including mountainous regions.

In the relative verification (i.e. based on N-value and AHD height differences), AUSGeoid09-based processing identified more outliers, but this can be explained by standard deviations generally being substantially smaller than those for the AUSGeoid98-derived results. AUSGeoid09 residuals were better distributed and consistently smaller than AUSGeoid98 residuals, which was particularly evident for large height differences. It was found that AUSGeoid09 generally provides AHD height differences at the ± 0.05 m to ± 0.09 m level (1 sigma) in the two study areas. Importantly, for practicing surveyors, the use of AUSGeoid09 has substantially increased the percentage of GNSS-derived height differences meeting third-order differential levelling specifications, with up to 90% of AUSGeoid09-derived height difference residuals falling within the maximum allowable misclose. This is a very encouraging result, considering the well-known difficulties of spirit levelling in mountainous terrain and the increasing popularity of GNSS-based height transfer in practice.

It is important to note that the results presented do not provide a general verification of AUSGeoid09 in other mountainous regions. It is acknowledged that sources of error exist within extensive datasets, even after careful investigation. Furthermore, GNSS and AHD height data have their own error budgets, and the sparseness of gravity data and any uncorrected biases in the spirit-levelling data in mountainous terrain may have contributed to some of the trends shown. However, considering that the main use of AUSGeoid09 is to compute AHD heights from GNSS-derived ellipsoidal heights, the data and method employed in this paper represent the most practical means of quasigeoid verification currently available.

The positive results of AUSGeoid09 performance in mountainous regions are encouraging, particularly in light of GNSS technology and CORS networks being increasingly used to provide vertical control. It can be expected that AUSGeoid2020, in conjunction with GDA2020, will provide further improvements for GNSS users, owing to improved modelling (including terrain modelling) and larger input datasets. To this end, DFSI Spatial Services continues to collect extended GNSS datasets on levelled marks across NSW (e.g. Gowans et al., 2015; Gowans, 2017; Janssen, 2017). The methodology presented in this paper will be very beneficial in regards to future testing of AUSGeoid2020. Eventually, however, the introduction of a new national vertical datum for Australia will be necessary in order to achieve higher consistency and generate a vertical reference surface that is more closely aligned to the geoid.

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