The Transit of Venus and 18th Century Positioning and Navigation

Case Bosloper
Retired Senior Surveyor, formerly Land and Property Information (Bathurst)
case.bosloper@bigpond.com

ABSTRACT

This paper outlines the significance of the Transit of Venus observations, as viewed through the eyes of a surveyor. Did the Nautical Almanac tables of contemporaries of First Fleet astronomer William Dawes incorporate any of the new information resulting from the transit observations of that century? The author presents an overview of the state of celestial mechanics of the 18th century and the exciting mathematical developments that were taking place at the time. A closer look is taken at how the predicting of the eclipses of the satellites of Jupiter and the development of lunar theory was affected by the gaining of an understanding of the size of our solar system. Hand in hand with this were the technological improvements in the optics of instruments like telescopes that took place in Great Britain, and the progress in the cataloguing of star positions for navigation. On the continent, vast theoretical development in celestial mechanics took place in response to prizes offered by the French Academy of Science. The understanding of the way a gravity field looked in the presence of three orbiting attracting bodies not only influenced the understanding of the irregularities in the motions of the Moon, but also the understanding of the resonance of the motions of the satellites of Jupiter. In the case of the Jovian satellites, the observations of the apparent satellite eclipse times could only be understood in a mechanical sense after the proper light-time corrections could be made, and the data cleaned of this effect. The observing of the Transit of Venus played its role to bring this about.

KEYWORDS: *Solar parallax, celestial mechanics, Transit of Venus, satellites of Jupiter, history of navigation.*

1 INTRODUCTION

The two recent Transit of Venus events, in 2004 and 2012, have been of interest to surveyors, trained as they are in the principles of positioning and navigation. During the 18th and 19th centuries, the use of field astronomy was a common technique for a surveyor engaged in positioning, as it was right up to 30 years ago. Earlier Transit of Venus events have had an interesting history. Transits presented an opportunity to calculate the size of the solar system, as a global effort. At an inferior conjunction, Venus passes between the Earth and the Sun. These conjunctions of Venus can line up with alternate nodes of the planet's orbit and transits of Venus across the Sun then occur in an 8-year pair at each node, about a century apart.

For today's surveyors, the early Transit of Venus efforts can evoke the following question: How soon did the new solar system parameters resulting from the Transit of Venus observations of 1761 and 1769 influence the then state-of-the-art of our industry of positioning? This question unfolds into various other questions, as certain astronomical tables were necessary for positioning and navigation. How good were these tables at the time? How much did the transit results improve the tables for immersions of satellites of Jupiter, by the end of the 18th century? (When a Jovian satellite moves into the shadow of Jupiter, the word immersion is used.) What improvements in celestial mechanics were happening at the time? Did the tables of lunar distances benefit from any advances in this? The accuracy of navigation depended directly on the accuracy of these tables. The author has reviewed such advances made and checked some of the relevant navigational tables of this period.

2 THE IMMERSIONS OF THE SATELLITES OF JUPITER

Let us set the scene with a summary of a letter written to the Royal Society of London by James Hodgson in the first quarter of 1735 about immersions of Jovian satellites (Figure 1).

IV. *An Account of ſome Obſervations of the* Eclipſes *of the* Firſt Satellite *of* Jupiter, *compared with the Tables, by the ſame.*

James Hodgson

Phil. Trans. 1735-1736 **39**, 15-16

Figure 1: Title of James Hodgson’s letter to the Society, and a portrait of ‘Jacobus Hodgsonus’ (engraved by George White, MacDonnell Collection).

James Hodgson, who lived from 1672 to 1755, was master of the Royal Mathematical School in Christ’s Hospital in London, and was a member of the Royal Society. Hodgson wrote to the Royal Society in 1735 that he had reviewed half a century of Jovian satellite observations covering a period from 1677 to 1731 (Hodgson, 1735). Immersion tables of these ‘Galilean’ satellites were used for determining longitude on land after 1650. Immersion predictions in these tables were shown to the second (the French showed them to the minute). Hodgson said that he compared the reported observations of immersions of the satellites of Jupiter with the Flamsteedian tables from which the catalogue of immersion predictions was deduced. He found that the 244 Jovian eclipses observed in that time slot could be categorised as follows:

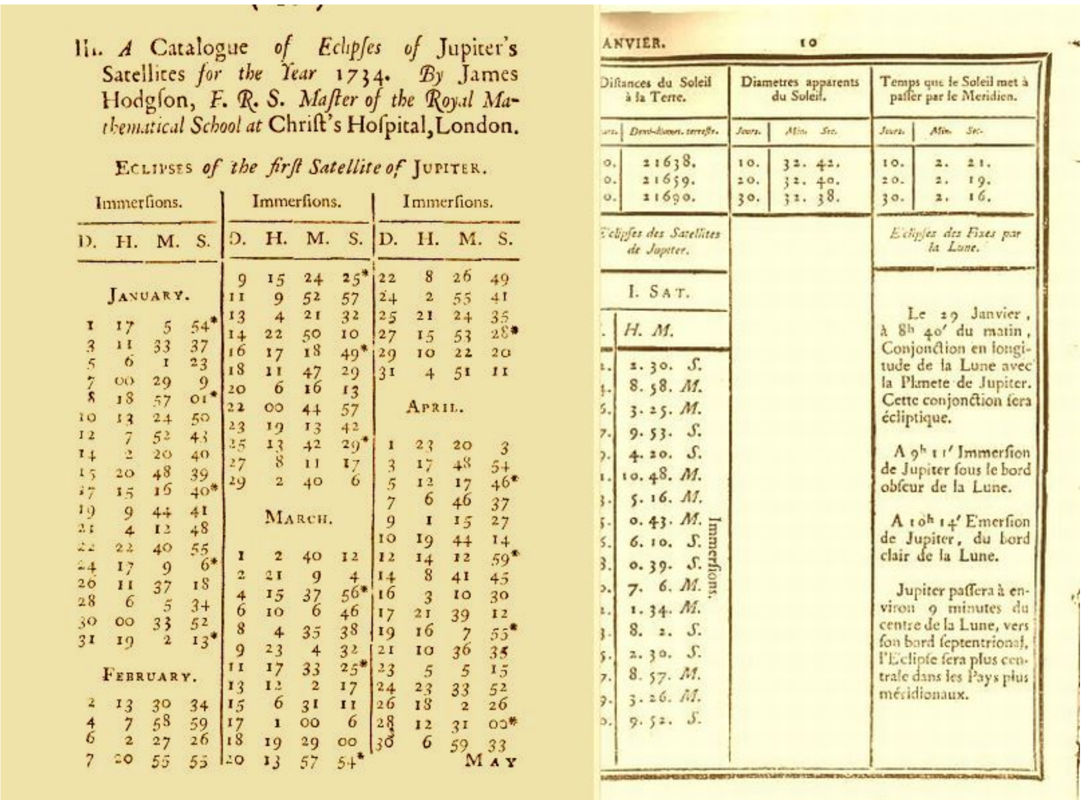
- 74 observations differed by less than 1 minute with the tabulated times,
- 53 differed between 1 and 2 minutes,
- 54 differed between 2 and 3 minutes,
- 33 differed between 3 and 4 minutes,
- and another 30 observations differed between 4 and 5.5 minutes from the published tables.

Hodgson used the generalised word ‘eclipses’. An error of a minute of time will result in an error of about 25 km in longitude at the equator. The worst difference between observed and predicted immersion events, by 1735, shown by James Hodgson, is 5.5 minutes of time. This is equivalent to a remaining 2/3 Astronomical Unit (AU) light-time correction, unaccounted for, if this would be the reason for the difference. Not having sufficient knowledge of the orientation of the line of apsides of Jupiter’s orbit (apsides are the points of greatest or least distance of the orbit of a celestial body from a centre of attraction) could also explain quite a proportion of this difference, as in Jupiter’s case the eccentricity is 4.8% of 5.2 AU. Today the speed of light is known, as is the size of the solar system. It was 1676 when Ole Roemer (1644-1710) saw the need for a light-time correction because not allowing for a finite speed of light had an accumulative effect. Roemer estimated that light took 11 minutes to cross one AU; now the accepted value is 8 minutes. The Jovian satellite Io makes 17 revolutions around Jupiter per month for example, so even an error of half a minute per revolution was going to compound. The interval between immersions appeared to decrease when Earth approached Jupiter, complicating the issue.

The errors in the tables were small enough though, so that someone could observe the immersion at about the right time. The trick would have been to start observing about half an hour early. If the same immersion were also observed simultaneously in another place with known longitude, it would be very useful to bring the observations together. This would allow longitude to be produced through post-processing by differencing. This cancels the effect of the accumulative prediction error. Thus, a multitude of Jovian satellite immersion observations in the British Empire was routinely sent off to Greenwich after reduction.

In the example of Figure 2, it is evident that the British Jovian satellite table of 1734 was in the old Julian calendric system; Britain's changeover to the Gregorian calendar was not until 1752. Here it should be noticed that:

- Hodgson's January 1 is Godin's January 12.
- The Greenwich times will be 9 minutes less than the Paris ones, to allow for the different meridians used.
- The Greenwich Mean Time (GMT) day started at noon (Hodgson's table), while the Parisian day starts at midnight.
- The 1st of January at 17h 05m 54s in Hodgson's table is the 13th of January 05h 16m 'matin' in Godin's table.



II. *A* Catalogue *of Eclipſes of* Jupiter's Satellites *for the Year* 1734. *By* James Hodgſon, *F. R. S. Maſter of the Royal Mathematical School at* Chriſt's Hoſpital, London.

ECLIPSES *of the firſt Satellite of* JUPITER.

Immerſions.			
D.	H.	M.	S.
JANUARY.			
1	17	5	54*
3	11	33	37
5	6	1	23
7	00	29	9
8	18	57	01*
10	13	24	50
12	7	52	43
14	2	20	40
15	20	48	39
17	15	16	40*
19	9	44	41
21	4	12	48
22	22	40	55
24	17	9	6*
26	11	37	18
28	6	5	34
30	00	33	52
31	19	2	13*
FEBRUARY.			
2	13	30	34
4	7	58	59
6	2	27	26
7	20	55	55

Immerſions.			
D.	H.	M.	S.
9	15	24	25*
11	9	52	57
13	4	21	32
14	22	50	10
16	17	18	49*
18	11	47	29
20	6	16	13
22	00	44	57
23	19	13	42
25	13	42	29*
27	8	11	17
29	2	40	6
MARCH.			
1	2	40	12
2	21	9	4
4	15	37	56*
6	10	6	46
8	4	35	38
9	23	4	32
11	17	33	25*
13	12	2	17
15	6	31	11
17	1	00	6
18	19	29	00
20	13	57	54*

Immerſions.			
D.	H.	M.	S.
22	8	26	49
24	2	55	41
25	21	24	35
27	15	53	28*
29	10	22	20
31	4	51	11
APRIL.			
1	23	20	3
3	17	48	54
5	12	17	46*
7	6	46	37
9	1	15	27
10	19	44	14
12	14	12	59*
14	8	41	45
16	3	10	30
17	21	39	12
19	16	7	55*
21	10	36	35
23	5	5	15
24	23	33	52
26	18	2	26
28	12	31	00*
30	6	59	33
MAY			

ANVIER. 10

Diſtances du Soleil à la Terre.		Diametres apparents du Soleil.		Temps que le Soleil met à paſſer par le Meridien.	
Jours.	Demi-diam. terreſtr.	Jours.	Min. Sec.	Jours.	Min. Sec.
0.	21638.	10.	32. 42.	10.	2. 21.
0.	21659.	20.	32. 40.	20.	2. 19.
0.	21690.	30.	32. 38.	30.	2. 16.

Eclipſes des Satellites de Jupiter.

I. SAT. — Immerſions.

H. M.
2. 30. *S.*
8. 58. *M.*
3. 25. *M.*
9. 53. *S.*
4. 20. *S.*
10. 48. *M.*
5. 16. *M.*
0. 43. *M.*
6. 10. *S.*
0. 39. *S.*
7. 6. *M.*
1. 34. *M.*
8. 2. *S.*
2. 30. *S.*
8. 57. *M.*
3. 26. *M.*
9. 52. *S.*

Eclipſes des Fixes par la Lune.

Le 29 Janvier, à 8h 40′ du matin, Conjonction en longitude de la Lune avec la Planete de Jupiter. Cette conjonction ſera écliptique.

A 9h 11′ Immerſion de Jupiter ſous le bord obſcur de la Lune.

A 10h 14′ Emerſion de Jupiter, du bord clair de la Lune.

Jupiter paſſera à environ 9 minutes du centre de la Lune, vers ſon bord ſeptentrional, l'Eclipſe ſera plus centrale dans les Pays plus méridionaux.

Figure 2: Example of a tabulation of Jovian satellite immersions for the meridian of Greenwich, for the satellite Io, by Hodgson, with at the right a January page from 'Connaissance des Temps' for the meridian of Paris, by Godin (M=matin, S=soir). The French page uses the Gregorian calendar, as adopted in 1582. Hodgson's dates are in the old Julian calendar, before the now 11-day adjustment to the Gregorian one.

Elizabeth I had been convinced by her scientists to go along with the changeover to the Gregorian calendar in 1582, but the English had an issue with the number of days to be intercalated. The English wanted to intercalate 11 days in 1582; Pope Gregory XIII's papal bull had proposed adjusting the calendar by 10 days, the excess of leap days since the Council of Nicaea. As a more sensitive matter, the Spanish attempted an invasion of England in 1588 with support from the next pope, of all things. This put an end to the matter of course, and Britain stayed with the Julian calendar for another 170 years (Duncan, 1999).

Giovanni Cassini (see section 6.1) and Jean Picard already struggled around 1671 with the question of Jovian satellite immersion prediction errors, but could not explain it. General acceptance of Ole Roemer's explanation regarding the finite speed of light took until 1727, when Astronomer Royal James Bradley (1693-1762) made his measurements of stellar aberration and also determined that the mean Sun to Earth light-time distance of one AU was 493 seconds (the currently accepted value is about 498 seconds). The French 'Connaissance de Temps' almanac of 1734 carried the suggestion to observe one immersion and apply the prediction error to the next tabulated values. So how good were the nautical almanacs that came after 1765, after Nevil Maskelyne became Astronomer Royal, almost a century after Ole Roemer's discovery (Figure 3, left portrait)? What was still lacking in the celestial mechanics (also called astrodynamics) of that age, to enable improvement of the almanacs?

The above questions are addressed in the timeframe of the latter half of the 18^{th} century, in the context of how well the tables served Australia's First Fleet astronomer William Dawes in Sydney. This organises the investigation by anchoring it somewhere in time. Dawes regularly observed immersions of Jovian satellites. The nautical almanacs of the 1780s were used for the lunar distance method of determining longitude at sea; and for longitude on land one used the tabulations of the times of the immersions of Jovian satellites, tabulated to the second in GMT. Dawes was issued with the nautical almanacs covering the years 1787 to 1792, so they were obviously printed 5 or 6 years ahead of the current date. This means the 1787 almanac was probably printed before or in 1781, and only reflected the knowledge of the 1770s.

In order to generate these Jovian satellite tables today, one needs a good understanding of the celestial mechanics involved in the mutual orbital resonance of these bodies, while embedded in the gravity field of each other and of Jupiter as well as of the Sun and planets. This is at least a three-body gravity problem, or even a 'four-or-more-body' gravity problem. Once one allows for a light-time correction, the phenomena can be time-tagged better and it becomes possible to separate this effect from the observed data. Only then, a theoretical framework of celestial mechanics can start emerging, with accuracy.

In the case of the Jovian satellites, other forces of disturbance were due to Jupiter's flattening and both Saturn's and the Sun's gravity. The interplay of the Jovian satellites could only be calculated by differencing and fitting polynomia to observed events, similarly as was done with the Moon. Pehr Wargentin (1717-1783) from Stockholm Observatory (Figure 3, right portrait) generated Jovian satellite eclipse predictions quite successfully in this way, but this also still had an incomplete theoretical framework. Wargentin published his first paper on the Jovian satellites in 1741 in the Transactions (Acta) of the Royal Society of Sciences in Sweden. Wargentin's tables were published by J. De Lalande in 'Connaissance des Temps', and after 1765 Nevil Maskelyne inserted these into his nautical almanacs. In the tables after 1746, Wargentin, who was now adjunct professor of astronomy at the University of Uppsala, did allow for what was called 'the great inequality' of the second Jovian satellite of 437.6

days (a periodicity found by differencing of differences). He attributed the effect to the mutual gravitational attraction of the satellites but did not have an analytical derivation for this (De Sitter, 1931). Through analysis of the multiple three-body problems, the satellite interactions are understood as orbital resonance today. When one moon gets ahead of schedule, another one seems to pull it back.

Figure 3: Ole Roemer, left (Frederiksborg Museum), and Pehr Wargentin, right (Svenska Familj-Journalen).

3 THE ORBIT OF THE MOON

In order to generate lunar distance tables, a good understanding is required of the celestial mechanics involved in the lunar motion and the rate of variation of its orbital elements while subject to the gravity field of the Sun and planets. This is a three-body gravity problem and even a more multiple body gravity problem when the other planets are included. A light-time correction is hardly relevant here as it amounts to only one second. The biggest questions for the development of a theoretical framework of the lunar motion involved the bothersome and little understood change in the variations of the orbital elements of the lunar orbit.

Apart from agreeing only approximately with the two-body Newtonian equations, the Moon's longitude was affected by phenomena like lunar evection and lunar variation. Evection means the eccentricity of the lunar orbit changes depending upon the orientation of the line of apsides with respect to the Sun. Although Ptolemy (90-168 AD) coped with evection via his epicycles and deferents, what lacked was a means to deal with this in a new mathematical sense, using celestial mechanics. What Ptolemy did was equivalent to performing a polynomial fit to observed data, without a theoretical framework. Later, Tycho Brahe (1546-1601) observed (actually rediscovered) the lunar variation in 1590, another lunar longitude effect eventually explained later by Lagrange as also caused by the Sun's gravity acting as a disturbance force on the lunar motion around the Earth. This leads to apparent accelerations and decelerations of the lunar motion, separate from the well-known difference between true and mean anomaly resulting from the elliptic orbit. Newton did treat the three-body problem in his 'Philosophiæ Naturalis Principia Mathematica' (the 'Principia'), but without giving the solutions to the 8^{th} order equations. A rather complicated gravity field landscape had unfolded here that was not properly dealt with yet. This had to wait until the three-body gravity problem was sufficiently developed analytically.

When the Moon's perigee is at quadrature to the line from the Earth to the Sun, the direction of the rate of rotation of the line of apsides (which contains the perigee and apogee) becomes opposite compared to the direction of the rate of rotation of the line of apsides when the Moon's perigee lines up with the line from the Earth to the Sun. Therefore, it lurches each way, yet on average the Moon's line of apsides rotates a little in excess of 40° per year in an anti-clockwise direction, relative to the line from the Earth to the Sun. An explanation for these variations was still awaited (a first tentative solution only came in 1749). Exacerbating this, the line of nodes (which contains the ascending and descending nodes) rotates clockwise by a little over 19° per year, but irregularities in its rate of motion also occur when the line of nodes is oriented at quadrature to the line from the Earth to the Sun. This rotation of a little over 19° per year explains the lunar cycle of 18.6 years, known to the ancients. These issues were important for predictions for the lunar distance method.

Therefore, in the context of the transits, at least three important things were happening in 18th century astronomy. One was the development of insight in the finite speed of light and the acceptance of this. The second was the use of the solar parallax in order to get an accompanying insight into the real size of the solar system. The third was the development of a better analytical or numerical understanding of celestial mechanics. It should be noted that the latter was influenced by the first two.

4 IMPROVEMENTS OF THE MEASUREMENT TOOLS

In order to make these developments possible, the measurement tools had to be improved. Christiaan Huygens (see section 6.1) solved the problem of chromatic aberration (Gribbin, 2003) in 1662, but this created more spherical aberration (Watson, 2004). The Dollonds found solutions perfecting the observing instruments and they could reproduce these in quantity, e.g. the John Dollond double object glass achromatic telescopes made in 1758 (Figure 4) and his son Peter Dollond's even better treble object glass (apochromatic) telescopes made in 1763 (Figure 5). These were also good for observing the moons of Jupiter.

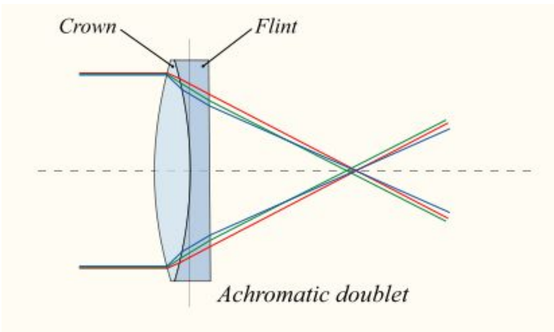


Figure 4: Achromatic doublet lens (left) and John Dollond (Royal Museums Greenwich) (right).

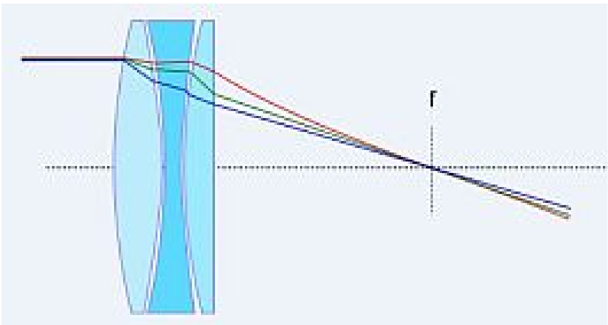


Figure 5: Triple achromatic (or apochromatic) lens (left) and Peter Dollond (Royal Museums Greenwich) (right).

The other development was the improving of the catalogue of star positions, especially in a

12° band centred on the ecliptic, for measurement of lunar distances. Of interest is that William Dawes was issued with the double object glass achromatic telescope. He requested the treble object glass one, owned by the Board of Longitude, but his request was unsuccessful as that was Nevil Maskelyne's favourite instrument.

Methods of observing and equipment specifications were also important. In 1773, Jean-Sylvain Bailly (1736-1793), a French astronomer and mathematician from Paris, showed the influence of aperture on the timing of an immersion. He had masked the aperture of his telescope down to a smaller aperture (17 Parisian lignes or 38.3 mm) when observing a Jovian satellite immersion. With this reduced aperture, he observed a Jovian satellite lose all its intensity and disappear. Taking away the aperture mask, he could again see the satellite for another two or more minutes through the full and larger aperture (of 24 Parisian lignes or 54.1 mm). The significance here is that when Jupiter is 40% further away than before, e.g. nearer conjunction rather than opposition, the light intensity of the Jovian moons will be 1.4 x 1.4 less than before, as intensity relates to the square of the inverse distance. Just this intensity effect will already cause a difference between observed and predicted time.

In addition, if the immersion times observed in various places are compared in order to deduce longitude differences, it would be of advantage to have similar apertures, or one has to make use of an aperture-dependent correction value. Bailly reasoned that if two telescopes with different apertures would be observing the time of the satellite immersion, the telescope with the smaller aperture would appear to see the immersion happen a couple of minutes too early, as the telescope with the larger aperture would continue to see the uneclipsed moon even minutes later. The necessary aperture correction value would use a calculation of the actual part of the satellite disc that could still be uneclipsed at the instant when the light intensity drops below the threshold of the telescope of a certain aperture. Bailly thought it useful if everyone recorded this limiting aperture during their observation.

It is worth mentioning some of the developments with micrometers. The first fixed micrometer with an invariable scale was credited to Christiaan Huygens (see section 6.1). Giovanni Cassini invented the reticulum, the oblique wire micrometer, for measuring differences in right ascension and declination, this micrometer was later improved by James Bradley. Ole Roemer had suggested a double image micrometer in 1675, but the idea was lost. Servington Savary independently developed one in 1743, by introduction of a split element into the optical path, producing a double image. In 1753, John Dollond combined Savary's divided object glass with a new method of measurement by Bouguer and came up with the divided object glass micrometer.

5 THE SOLAR PARALLAX

The parallax effect is what makes our eyes perceive distance. The use of parallax for astronomy was understood more than 2,200 years ago. It is said that Aristarchus of Samos (310-250 BC) noticed on a sundial that when it was exactly first quarter of the Moon, the Moon was not at 90° to the Sun. He estimated the angle to be 1/30 smaller than 90° and realised the Sun was at least 20 times as far as the Moon (Dreyer, 1953). He actually wrote a book 'On the Dimensions and Distances of the Sun and Moon' although he did not pursue the subject much further. Hipparchus of Nicaea (190-120 BC), famous for his discovery of precession of the equinoxes, became aware that the March 14 solar eclipse of 189 BC had looked different to observers in widely separated places (Hirshfeld, 2001). This is a parallax effect. The eclipse was total in the Dardanelles (the Hellespont) but the Moon was seen as only covering the Sun by four fifths in Alexandria. Knowing the latitudes of these places, Hipparchus, after Eratosthenes' work regarding the Earth's circumference, figured the Moon to be 35 to 40 earth diameters away from Earth. We now know the correct value is 30.

Parallax measurements are still an important tool today. One example illustrates a transit method of the 1970s. When the first artificial satellites were orbiting Earth after 1957, it was soon realised that the parallax effect was not only a way to track the satellites but also a very efficient way to do transcontinental triangulation of European observatories. The idea of a bundle adjustment comes to mind. The satellite was photographed simultaneously from different observatories against the backdrop of stars (Figure 6, top). Time tags were inserted into the satellite track being photographed, using a louvre shutter action on the telescope that was timed to milliseconds, which created repeated mid-exposure dot-like gaps in the satellite trace. The right ascension and declination of these gaps in the trace were measured in a comparator, in relation to the nearby star images. This produced time tagged satellite locations. It should be noted that the stars are regarded as being an infinite distance away.

When all the photographs of the different observatories were measured and the data centrally collected, one could solve for corrections to the numerically integrated predicted satellite state vector and corrections to the initial positions of the observatories. This way one could triangulate in giant strides across the continent and across the British channel, to about 5 parts per million (ppm), using the principle of parallax by way of satellites. The 30 m diameter aluminium coated Mylar balloon PAGEOS, launched in 1966 to an orbit between 3,000 and 5,000 km altitude, and the similar balloons GEOS-2, Echo-2 etc. where used in this way. In 1970, the author participated at one of the observatories and measured its photo plates.

Halley had pointed out that the Transit of Venus presented an opportunity to measure the size of the solar system. In 8 years (bar 2.4 days) Venus orbits the Sun almost exactly 13 times, so it would have overtaken the Earth 5 times. The positions along Earth's orbit where this overtaking occurs are neatly spaced at 72° intervals along the zodiac, one fifth of 360°. At these points a line-up of the Sun, Venus and the Earth occurs, familiar to us as the points where Venus disappears as an evening star and emerges as a morning star. These 5 overtaking points form a 5-spoke wheel of what we call inferior conjunctions. This wheel of conjunctions slowly rotates along the zodiac in a clockwise direction by a little less than 2.4° per 8 years, due to the small 2.4-day mismatch mentioned above. With slow regularity, one of the positions of inferior conjunctions occasionally lines up with one of the nodes of the orbit of Venus. This is where Venus' orbit intersects the plane of the Earth's orbit, the ecliptic plane. If we have an inferior conjunction at one of these places near the node, a transit can occur.

Behind the scenes, something else is happening at the other node: When a transit happens at one node, the opposite node of Venus' orbit will be exactly halfway between 2 of the other 5 spokes, halfway one of the 5 sectors of 72° and thus about 36° away from one of the inferior conjunction spokes. This means after about 15 sequences of 8 years (15 movements of 2.4° in 121.5 years) one will see a position of inferior conjunction again line up with a node, this time the opposite node. The window near the nodes is just wide enough to accommodate two transits, especially if the line-up with the node is not perfect. When this happens a transit can take place just before the node, 8 years early, and as the last transit at the other node was the second transit in the 8-year two-transit sequence and thus a little past the node, the gap between the transits is at times 105.5 years. The next transit double event is then 121.5 years away to complete the 243-year periodicity, with again a change of nodes. This also means the transits occur about once a century as a double event at the node near the June part of Earth's orbit, alternating with a double event at the node near the December part of Earth's orbit.

Projecting the planet Venus onto the body of the Sun (Figure 6, bottom) from widely separated points on Earth, and measuring its parallax is no more complicated than the above satellite example, in concept. In practice, it is different as the Transit of Venus happens in the daytime, so the measurements have to be made against the undulating edge of the body of the Sun (Figure 7).

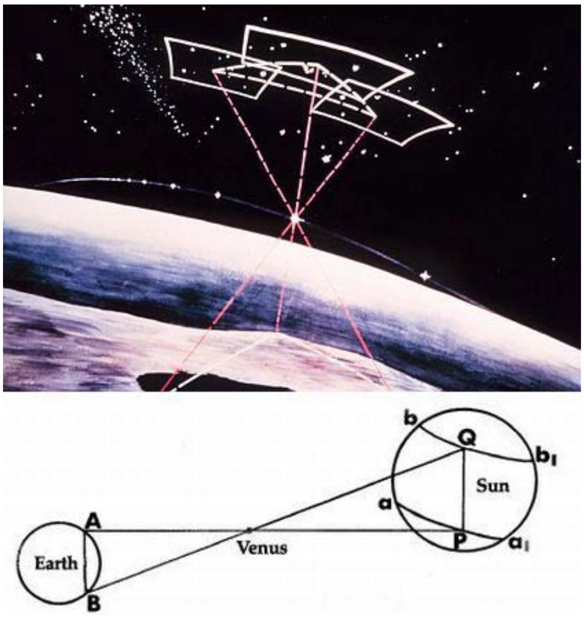


Figure 6: The principle of parallax applied to a 1960s satellite (NOAA Photo Library) (top) and parallax applied to Venus after Howse (1969) (bottom).

In this case, the Sun is at a finite distance, which has to be solved for. This undulation complication, still valid for modern observers today, was expressed by Pehr Wargentin (who observed the 1761 transit) by reporting to the Royal Society, in Latin (Wargentin, 1761): "*Venus jam aliqua sui parte discum Solis occupaverat. Propter vehementem marginum Solis undulationem, primum contactum exteriorem accuratius notare non potui.*" Freely translated this equates to "Part of Venus itself was already to some extent covering the solar disc [when I timed it]. Because of vigorous undulation of the solar margin, I could not accurately record the first exterior contact."

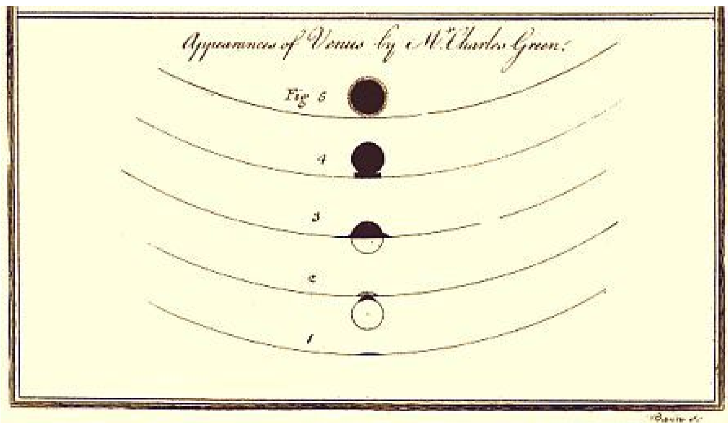


Figure 7: Drawing of the exterior contact (1) and interior contact (5) at the 1769 ingress (Green, 1771).

It is worth noting that this was also exactly the author's experience when timing the ingress of the 2004 Transit of Venus at a re-enactment at Woodford NSW. Gary Hovey from Mt Stromlo observatory was present, who supplied the UTC 1-second time pips with an early breadboard version of the VNG users' consortium GPS time receiver, after the VNG radio time signal broadcast ceased to operate in 2002 (Hovey and Herald, 2005). The interior contact was a lot easier to observe, when compared with the exterior contact.

In 1763, the solar parallax (the angle subtended at the Sun by the Earth's mean radius) from the 1761 Transit of Venus based on 53 observatories had been determined as 8.56", but it was acknowledged that there was disagreement between various methods, some even resulting in 10.5". In 1771, Thomas Hornsby produced a summary of the next solar parallax results (from the 1769 transit) in a letter to the Royal Society. At the end of the letter he showed a table that listed the Astronomical Unit as having a length of 93,726,900 English miles (Table 1). The metric equivalent is 150.8 million kilometres, which is within 1% of the currently accepted value of 149.5 million kilometres and 100 years after Cassini had first used the parallax of Mars for this. Hornsby (1733-1810) was a mathematician and astronomer, and a fellow of the Royal Society. After 1763, he occupied the Savilian Chair of Astronomy at Oxford. Thomas Hornsby observed the 1761 Transit of Venus in Oxfordshire.

Table 1: Solar system dimensions in English miles, calculated by Hornsby (1771).

	Relative diſtance.	Abſolute diſtance.
Mercury,	387,10	36,281,700
Venus,	723,33	67,795,500
Earth,	1000,00	93,726,900
Mars,	1523,69	142,818,000
Jupiter,	5200,98	487,472,000
Saturn,	9540,07	894,162,000

Oxford, Dec. 17, 1771.

6 DEVELOPMENTS IN CELESTIAL MECHANICS

The Paris Royal Academy of Sciences has played a prominent role on the continent towards encouraging the development of the necessary mathematics for orbital mechanics. In 1788, Sir Henry Charles Englefield (1752-1822), an English scientist who was elected a Fellow of the Royal Society in 1788 at the age of 26, wrote a lamenting passage in an introduction to his book (Englefield, 1788) 'Tables of the apparent places of the comet of 1661, whose return is expected in 1789' (yes, that is William Dawes' comet). He said that since the death of Edmund Halley in 1742, practically nobody in England had written anything substantial on the science of orbits. Although this passage was written in the context of comets, he points to the work done in Germany, France and Russia by people like Lambert, Clairaut, Lagrange, Laplace and Euler, who all also played a wider role in other aspects of celestial mechanics.

6.1 The Founding of the Paris Royal Academy of Sciences

The early development of solutions for the equations of triple-body and multiple-body celestial mechanics owes a lot to the Paris Academy of Sciences. Jean-Baptiste Colbert (the French First Minister, Minister of Finances, later the Secretary of State of the navy) had suggested the value of royal patronage for science to King Louis XIV. Colbert had already gathered a group of eminent scholars, which also happened to include the 'géomètre' Christiaan Huygens (1629-1695). Christiaan was a Dutchman educated at the University of Leiden, who later corresponded with contemporary scientists like Isaac Newton and played a role in the formulation of Newton's second law of motion.

Huygens was from a family that had held down the post of Secretary of State of the Stadholders of the Dutch Republic for three generations (Jardine, 2009). Christiaan's brother Constantine Huygens became the Secretary of State, just as his father and grandfather had been. The Huygens family saw to it that their sons were given the necessary wide-ranging, cultural and penta-lingual tertiary education, and also included in the character formation of their sons such useful things as training in classics, horsemanship, etiquette, diplomacy, music, song, poetry, politeness, composure and even posture, so they could move with confidence in international circles and courts (Stoffele, 2006). The Huygens brothers were trained in their teens to write polite letters in diplomatic language to each other, when they had to solve mundane domestic disagreements amongst themselves. Christiaan eventually showed a preference towards the sciences, after travelling for a brief period as a diplomat.

Aged 28, he published a book on probability theory in 1657. In the following years he formulated the centripetal force concept, became adept in the grinding of lenses, discovered in 1657 that Saturn had rings and explained them, showed that Saturn was not a contact triple planet as Galileo had described, discovered Saturn's moon Titan, derived the famous pendulum equation and was the first to measure precise gravity with this. He later contributed to the development of the 'moment of inertia' concept.

Christiaan already became a Fellow of the Royal Society of London in 1663, and was in Paris in 1666 when he saw that the French wanted to set up their own Académie Royale. The proposed members had not organised themselves yet, apart from some limited get-togethers of some of them in Jean-Baptiste Colbert's library. As he was a senior figure of the Parisian Montmor Academy and the only foreign scholar selected for the Royal Paris Academy by Colbert, Christiaan Huygens was perfectly suited to take on temporary leadership and initiate the 1666 inaugural meeting of the Paris Academy of Sciences. This was because he was not part of the competing vested interests. He was fluent in French and well trained for this formal function. Christiaan Huygens had a good relationship with the French King, was already in receipt of a pension of 1,200 livres from the King thanks to Colbert in 1663, and had earlier received the King's privilege in 1665 with respect to distribution in France of his pendulum clock (somewhat like a patent). Therefore, the Paris Royal Academy of Sciences was born in 1666. For the next three decades, it was referred to as L'Assemblée (Figure 8).

There are no minutes of the first meetings but some accounts are accessible (Sturdy, 1995). These show that Cassini (who joined three years after the inaugural meeting) and Huygens received a generous annual pension of about four times the size of what the other scientists received, reflecting Huygens' and Cassini's prestige. In the list of founding members of the Academy, Huygens is identified as a 'géomètre', the French word also used for surveyor, although he is mostly known as a mathematician, physicist, horologist and astronomer. Huygens left the Academy in 1682 and his extensive library of thousands of books, including a plethora of books on surveying among 300-odd mathematical books, was auctioned after his death in 1695. The Paris Royal Society also included Johannes Hevelius as one of its members. Eventually, it was for the return of one of the comets sighted by Hevelius in 1661 that William Dawes was sent to Australia more than a century later. It was 80 years after the inaugural meeting that certain biannual prizes offered by the Paris Academy became the catalyst for further important developments in celestial mechanics.

Figure 8: Colbert presenting the members of the French Royal Academy of Sciences to Louis XIV in 1667. Detail of a 1672 painting by Henri Testelin (Palace de Versailles).

6.2 Celestial Mechanics Prizes Offered by the Paris Academy

Celestial mechanics had intrigued people throughout the ages. Thales of Miletus (640-516 BC), who rejected mythological explanations of phenomena (and this became fundamental to the scientific revolution), had taught the sphericity of the Earth after being instructed in Egypt. Pythagoras (569-470 BC) taught that the Earth rotates and revolves. Aristotle (384-322 BC) maintained that the Earth was round. Aristarchus of Samos (see section 5) had championed the idea of a heliocentric solar system, but his idea did not gain acceptance as the conventional wisdom although his contemporaries such as the 20 years younger Archimedes (287-212 BC) were aware of his heliocentric view and are known to have discussed it. Even 300-odd years later, there was still awareness of the view of Aristarchus. Plutarch (46-120 AD), in 'On the face in the disc of the moon', says someone held that Aristarchus supposed "that the heavens stand still and the Earth moves in an oblique circle at the same time as it turns around its axis" (Dreyer, 1953). One generation later, Ptolemy did not adopt the heliocentric system. Another millennium went by.

It took until Copernicus' posthumous publication in 1543 about the heliocentric circles for planetary orbits, for the issue to regain a wider audience. The publication was posthumous in order to escape raised eyebrows from the Church.

Further insight was gained by Johannes Kepler (1571-1630), who benefited from being able to use the set of accurate observational data of his mentor Tycho Brahe. Kepler's laws of planetary motion of 1609 and 1619, which assumed elliptical orbits, allowed some analytical understanding of these motions. The planets apparently obeyed some very precise laws. These were later given physical substance by Newton's equations of mutually attracting masses, published in his 'Principia' in 1687. However, these solutions for planetary motion generally treated everything first as a two-body problem. The reality of the solar system, with a bit of exaggeration, is more comparable with half a dozen children jumping on a single trampoline, together with one very obese one, and all of them affecting the balance of the others (Figure 9). Newton did study the three-body problem, but his treatment awaited someone who could solve the complicated set of 8th order equations. Parts of Newton's unpublished work, like that about precession of the lunar perigee, were not discovered until 1872.

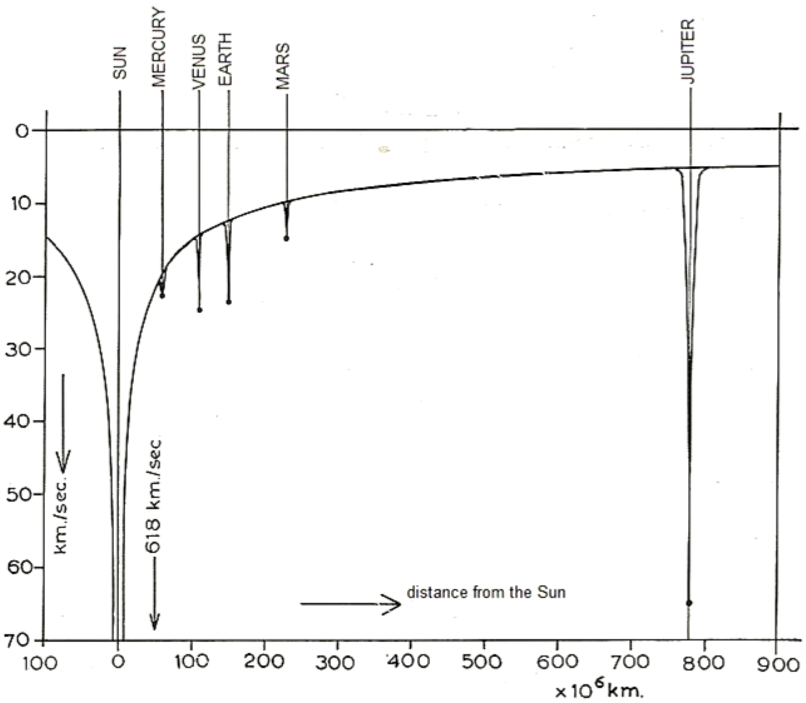


Figure 9: Escape velocity minus circular velocity, after Vertregt (1959), showing the interplay of multiple body gravity fields.

Some of the observed celestial phenomena did not conform to the conventional and usual two-body theory of gravity. These were called 'inequalities' of the phenomena. The inequalities of planetary and lunar motion led the Paris Academy of Sciences to propose that the solving of certain analytical problems should be subject to a competition. Those competitions led to important developments in celestial mechanics. In the period under consideration, Joseph-Louis Lagrange (1736-1813) played a prominent role among others like Leonhard Euler (1707-1783), Alexis Clairaut (1713-1765) and Jean-Baptiste le Rond d'Alembert (1717-1783), shown in Figure 10.

Figure 10: Leonhard Euler (by J.E. Handmann), Joseph-Louis Lagrange (wiki), Alexis Clairaut (by L.J. Cathelin) and Jean-Baptiste le Rond d'Alembert (The Louvre).

Only the Academy's competitions relevant to this story will be mentioned in this paper. Although Pierre-Simon Laplace has also been very important in this field, he was only born in 1749 and his influence covers a slightly later period. He published on Jupiter-Saturn perturbations in 1784-1786, and on lunar acceleration in 1787. The first part of his 'Méchanique céleste' was not published until 1799.

6.3 Lunar Theory and the Three-Body Problem

The demand for lunar tables was high due to their importance for navigation. Various learned societies offered substantial prizes for lunar tables that could be proved to agree with observations within narrow limits. Euler, a Swiss scientist, set the scene by publishing a set of lunar tables in 1746, but these were rather imperfect.

While Newton had a geometric approach in his perturbation theory, with respect to lunar motion, Clairaut and d'Alembert, two Parisians, made their advances through integration of differential equations. They sent memoirs on lunar theory to the Paris Academy in 1747, but still could not properly explain the rather irregular motions of the lunar perigee. Clairaut won a prize in 1752 set by the St Petersburg Academy, with 'Théory de la lune'.

Euler published more on lunar theory in 1753, and Tobias Maier (1723-1762) of Göttingen, Germany, compared this with observations. He saw that the eight unknowns which Euler was solving were sensitive to the choice of observations held in the eight simultaneous equations and systematically optimised the solution by combining equations and adding occultations of the star Aldebaran by the Moon. So successful were Maier's corrections to Euler's theory, that the English government offered Euler and Maier a payment of £3,000. The lunar theory had become accurate to one arc minute and became the basis of the nautical almanac of 1767 and later (Bradley and Sandifer, 2007).

In 1762, the Paris Academy established a competition for 1764 to explain why the Moon always shows the same face to the Earth (a tidal resonance effect) and whether it undergoes precession and nutation. Lagrange responded with his successful 'Recherches sur la libration de la lune' but failed to explain the strange motion of the line of lunar apsides and that of the line of lunar nodes.

In 1763, values for the size of the solar system were published, resulting from the 1761 Transit of Venus. This was followed in 1771 with results from the 1769 transit. It was then that the three-body problem became a subject of the competition in 1772. This resulted in Lagrange producing his groundbreaking 'Essai sur la problème des trois corps' in reference to the Moon. This later led to his development in 1788 of Lagrangian mechanics. One can imagine this as based on the analogy that water finds that path downhill that minimises the action that is required. In the same way, a body in a complicated gravity landscape (Figure 11) will find that path of least resistance or the least required action. In fact, in an elliptical trajectory, there is an energy balance where no energy needs to be added and no energy is withdrawn in order to continue the motion. If the potential energy increases, the kinetic energy decreases to measure. Without this energy balance approach, one would have to calculate the effect of gravity between each possible pair in three bodies at every single point along their path in order to determine that path. Lagrange shared this prize with Euler, who had already made a submission on the same subject with his 'Nouvelles recherches sur le vrai mouvement de la lune'.

In 1774, the Paris Academy sought an explanation for the secular equation of the Moon and whether that involved the gravity fields of all celestial bodies or whether the non-sphericity of Earth and Moon played a role herein. Lagrange's 'Sur l'équation séculaire de la lune' won the prize.

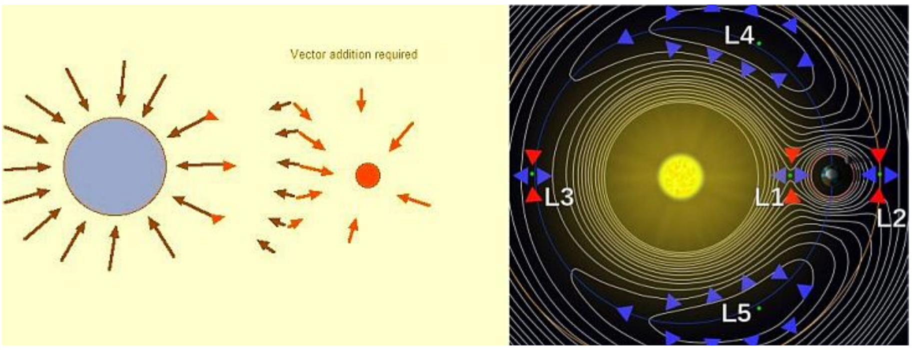


Figure 11: Addition of the gravity vectors of the two bodies are required, resulting in an interesting gravity landscape for a third body, like our Moon, to move through. This led to the discovery of the Lagrangian points.

When the mathematician William Rowan Hamilton (1805-1865) later developed his Hamiltonian mechanics in 1833, he acknowledged his debt to Lagrange's energy balance approach. No new solutions to the three-body problem were found until George William Hill (1838-1914) developed his lunar theory in 1878. Hill's solutions were of a substantially greater practical value than those of Lagrange. Hill's work was not surpassed until Henri Poincaré (1854-1912) took it further again in 1892 with a groundbreaking and more profound approach published in 'Les méthodes nouvelles de la mécanique céleste' (Moulton, 1970). He had also shown that the equations for the secular terms of lunar (and planetary) motion diverged rather than converged and this had consequences for the stability of the solar system. Poincaré's work later led to chaos theory and the Lyapunov exponent, which play a role in celestial mechanics.

6.4 Motions of Jupiter and its Satellites

Euler first had derived the differential equations for perturbations in general, and submitted this work to the Berlin Academy in 1747. Then Euler wrote a memoir, also in 1747, with the derivation of the perturbations upon Saturn by the action of Jupiter, and submitted it for the prize set by the Paris Academy of Sciences for 1748. Euler won the 1748 prize.

For 1766, the Paris Academy had put forward the question of what inequalities should be observed in the motions of the four Jovian satellites as a result of their mutual attractions. D'Alembert had earlier objected to the wording of this question, as it appeared to ignore the gravity of the Sun. Lagrange won the prize with 'Recherches sur les inégalités des satellites de Jupiter'. Euler followed in 1769 with 'Recherches sur les inégalités de Jupiter et de Saturne'.

After Lagrange's work in 1772 on the three-body problem with regards to the lunar motion, it became possible to derive analytically why satellite III will be at quadrature when the Jovian satellites I and II line up with Jupiter, and that satellite I will be in opposition when satellites II and III line up with Jupiter. Similarly, when satellites I and III line up, satellite II will be in opposition or in two other places of always the same fixed azimuth. Although this interplay of the satellites is a temporary resonance that unwinds in the long term because the equations of motion do not converge, it has lasted for centuries.

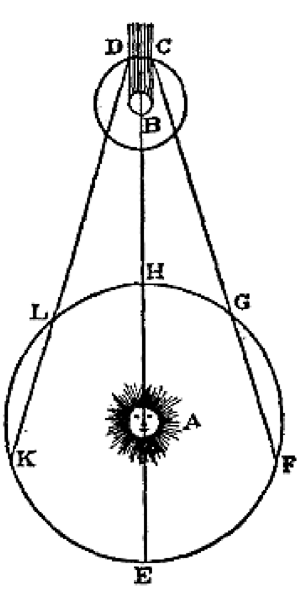
Lagrange's 1772 work on the three-body problem and the later 1788 development into Lagrangian mechanics still had to become widely known, understood and accepted, before it could start to affect the elaborate calculations for the nautical almanacs for Jovian satellite immersions.

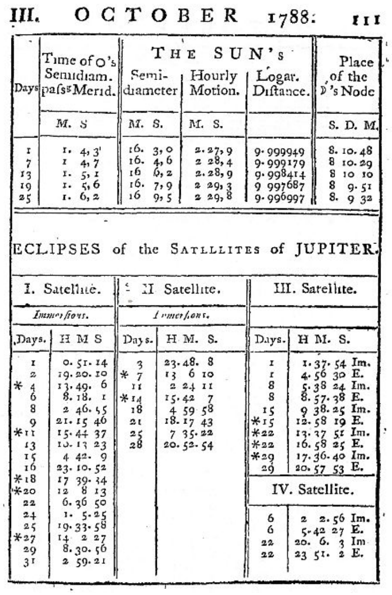
7 COMPARISONS OF TABLES OF JOVIAN SATELLITE IMMERSIONS

The 1787-1792 nautical almanacs carried by William Dawes were each printed half a decade or more in advance. There would be a small chance that some of the advances made in celestial mechanics just before 1781 would have found their way into those almanacs, but the lead-time involved in the required effort of calculation and preparation of the tables was enormous. In order to spot-check the quality of the Jovian satellite immersion tabulations, some comparisons will be made of those in the 1788 nautical almanac, with Jovian satellite phenomena generated by modern astronomical software.

From William Dawes' correspondence with Nevil Maskelyne (Morrison and Barko, 2009) it is known that Dawes observed the immersions of the Jovian satellite Io on 15 October, 7 November and 7 December 1788, for longitude. The October observation, by sheer coincidence, can be illustrated with a drawing by Ole Roemer, which shows a casual selection of points of the orbit of the Earth from which an immersion of a satellite of Jupiter is viewed (Figure 12). As the distance from Earth to Jupiter on 15 October 1788 was 5.2 AU, it can be compared with a point say 10° further than the point marked F, in an anti-clockwise direction on Earth's orbit. As seen from the Sun, the angle between the Earth and Jupiter was only a little more than a right angle. Three months earlier the distance to Jupiter was 6.2 AU (i.e. near E), and three months after October the distance was 4.3 AU (i.e. near H).

Of these Jovian satellite observations made in October, November and December 1788 the quality of the predicted times can be evaluated (Figure 12 shows October). A comparison can be made between tabulated values out of Dawes' 1788 nautical almanac and retrospective predictions from modern professional astronomical software like SkyMap Pro. When using SkyMap Pro 10 to check events that occurred in the past, like Charles Green's Jovian satellite immersion events in 1769 (Green, 1771) during his Transit of Venus observations, agreement is found within a few minutes of the 1769 immersion predictions. By applying this procedure to Dawes' observations from 1788, the results shown in Table 2 are obtained.





III. OCTOBER 1788: 111

Days	Time of ☉'s Semidiam. pass[g] Merid.	THE SUN's Semi-diameter	THE SUN's Hourly Motion.	THE SUN's Logar. Distance.	Place of the ☽'s Node
	M. S	M. S.	M. S.		S. D. M.
1	1. 4, 3	16. 3, 0	2. 27, 9	9. 999949	8. 10. 48
7	1 4, 7	16. 4, 6	2 28, 4	9. 999179	8 10. 29
13	1. 5, 1	16 6, 2	2. 28, 9	9. 998414	8 10 10
19	1. 5, 6	16. 7, 9	2 29, 3	9 997687	8 9. 51
25	1. 6, 2	16 9, 5	2 29, 8	9. 996997	8. 9 32

ECLIPSES of the SATELLITES of JUPITER.

I. Satellite. Immersions. Days.	H M S
1	0. 51. 14
2	19. 20. 10
* 4	13. 49. 6
6	8. 18. 1
8	2 46. 55
9	21. 15 46
*11	15. 44 37
13	10. 13 23
15	4 42. 9
16	23. 10. 52
*18	17 39. 34
*20	12 8 13
22	6. 36 50
24	1. 5. 25
25	19. 33. 58
*27	14 2 27
29	8. 30. 56
31	2 59. 21

II Satellite. Immersions. Days.	H. M. S.
3	23. 48. 8
* 7	13 6 10
11	2 24 11
*14	15. 42 7
18	4 59 58
21	18. 17 43
25	7 35. 22
28	20. 52. 54

III. Satellite. Days.	H. M. S.
1	1. 37. 54 Im.
1	4. 56 30 E.
8	5. 38 24 Im.
8	8. 57. 38 E.
15	9 38. 25 Im.
*15	12. 58 19 E.
*22	13. 37 51 Im.
*22	16. 58 25 E.
*29	17. 36. 40 Im.
29	20. 57 53 E.

IV. Satellite. Days.	H. M. S.
6	2 2. 56 Im.
6	5. 42 27 E.
22	20. 6. 3 Im
22	23 51. 2 E.

Figure 12: Jovian satellite immersions viewed from Earth on different months, after Ole Roemer (left) and the October predictions for Jovian satellite immersions in the 1788 nautical almanac (right).

Table 2: Jovian satellite immersion predictions of 1788, compared with modern astronomical software results.

Satellite immersion observed	Date	1788 Almanac	SkyMap Pro 10	1788 Almanac Offset	Distance to Earth
Io	15 Oct 1788	04h 42m GMT_{old} (16h 42m UT)	16h 26m UT	16m later	5.23 AU
Io	7 Nov 1788	04h 53m GMT_{old} (16h 53m UT)	16h 35m UT	18m later	4.87 AU
Io	7 Dec 1788	06h 46m GMT_{old} (18h 47m UT)	18h 37m UT	10m later	4.48 AU

The Earth had been travelling towards Jupiter from about 1 July 1788, so this makes the intervals between the Jovian satellite immersions for the next 6 months appear up to a quarter of a minute shorter per revolution than average as seen from the Earth. This apparent rotation period deficit is accumulative and makes the tabulated values appear late. It distorts the real times by a maximum accumulative 4 minutes per month after which it accumulates at a decreasing rate through a cosine factor. It appears that the difference already had accumulated to 16 minutes by October and was coming some way back towards the average by December. Some of this can be explained by the non-application of a light-time correction, and the balance obviously has something to do with unknowns remaining in other orbital variations like in the ‘equation of the centre’ of Jupiter, also known as the orbital eccentricity. The author finds that no light-time corrections appear to have been made to the Jovian satellite tables at that time, in the 1788 nautical almanac, more than a century after Ole Roemer’s explanation that light travelled at a finite speed.

8 CONCLUDING REMARKS

At the start of this paper, a few questions were asked about the possible early influence of the Transit of Venus results of 1761 and 1769. In answer to these questions, one can say that the Transit of Venus results did not yet improve the calculation of the values for the astronomical tables by the last decade of the 18th century. The tables for lunar distances were already of good quality at the time through inclusion of occultations of Aldebaran results into the calculations, but the increased knowledge about the variations in planetary orbital elements through understanding of the three-body problem came later. The tables of immersions of satellites of Jupiter appear not to have improved until the 19th century.

It can be concluded that tremendous improvements in celestial mechanics were happening during the 18th century, which have added a lot to our understanding of the mathematics and dynamics of orbital motion, in both the short term and the long term. Before the light-time correction was applied, anomalies still appeared to be present in the data as late as almost the 1790s. Of course, the data could be cleaned of this effect in the future. The main influence of the 1761 and 1769 Transit of Venus results during the 18th century was to give the eminent mathematicians some better ‘ground truth’ to check their theoretical derivations against while they were developing the theory, as such enabling their great advances at the time. The nautical almanac tables for the practitioners were not corrected until later.

REFERENCES

Bradley R.E. and Sandifer C.E. (eds.) (2007) *Leonhard Euler: Life, work and legacy*, Elsevier B.V., Netherlands.

De Sitter W. (1931) Jupiter's Galilean satellites (George Darwin Lecture), *Monthly Notices of the Royal Astronomical Society*, 91, 706-738.

Dreyer J.L.E. (1953) *History of astronomy from Thales to Kepler* (2nd edition), Dover Publications, New York.

Duncan D.E. (1999) *The calendar*, Fourth Estate Ltd., London.

Englefield H. (1788) *Tables of the apparent places of the comet of 1661, whose return is expected in 1789*, P. Elmsly, London.

Green C. (1771) Observations made, by appointment of the Royal Society, at King George's Island in the South Sea, *Philosophical Transactions of the Royal Society of London*, 61, 397-421.

Gribbin J. (2003) *Science: A history*, Penguin Books (Australia), Camberwell, Victoria.

Hirshfeld A.W. (2001) *Parallax*, W.H. Freeman and Company, New York.

Hodgson J. (1735) An account of some observations of the eclipses of the first satellite of Jupiter, compared with the tables of the same, *Philosophical Transactions of the Royal Society of London*, 39, 436-444.

Hornsby T. (1771) The quantity of the Sun's parallax, as deduced from the observations of the transit of Venus, on June 3, 1769, *Philosophical Transactions of the Royal Society of London*, 61, 574-579.

Hovey G. and Herald D. (2005) The VNG Users Consortium GPS time receiver, http://tufi.alphalink.com.au/vnguc/ (accessed Jan 2013).

Howse D. (1969) *Captain Cook's pendulum clocks*, Antiquarian Horological Society, London, 15pp.

Jardine L. (2009) *Going Dutch*, Harper Perennial, London.

Morrison D. and Barko I. (2009) Dagelet and Dawes: Their meeting, their instruments and the first scientific experiments on Australian soil, *Historical Records of Australian Science*, 20(1), 1-40.

Moulton F.R. (1970) *An introduction to celestial mechanics* (2nd revised edition), Dover Publications, Mineola, NY.

Stoffele B. (2006) Christiaan Huygens – A family affair, Master's thesis, Utrecht University, http://www.proevenvanvroeger.nl/eindopdrachten/huygens/huygensfamily.pdf (accessed Jan 2013).

Sturdy D.J. (1995) *Science and social status: The members of the "Académie Des Sciences", 1666-1750*, Woodbridge, Suffolk, UK.

Vertregt M. (1959) *Grondbeginselen van de ruimtevaart*, De Erven F. Bohn N.V., Haarlem. [in Dutch]

Wargentin P. (1761) Observations on the same transit; and on an eclipse of the Moon, May 8, 1761, and of the Sun, on the 3d of June 1761, *Philosophical Transactions of the Royal Society of London*, 52, 208-213.

Watson F. (2004) *Stargazer: The life and times of the telescope*, Allen & Unwin, Crows Nest, NSW.