Selecting the optimum spacing for digital elevation models

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Abstract

Digital Elevation Models are likely to be created more and more often by surveyors in the future, instead of hard copy forms of depicting topography by contours plots. A digital elevation model - a set of heights on a regular grid to depict topography - is easily manipulated on a computer to create an effective representation of surface for viewing on a screen, for hydrological modelling, and so on. However, it can be difficult to select the single grid spacing which is suitable to cover an entire survey area. There are some ways that people try to select a suitable grid spacing before the survey is carried out. Failure to select the optimum grid spacing can cause errors, especially if volumes are calculated.

Digital Elevation Models

A digital elevation model is simply a set of coordinates of points across a surface. It is probably better to think of a DEM as a set of horizontal coordinates, each one with an associated height coordinate (but no detail code). A DEM is usually regarded as having the points distributed on a regular grid, such as

17	14	-1695	
18	14	-1305	
19	14	-1277.5	
20	14	-1345	
21	14	-1425	
22	14	-1502.5	
23	14	-1490	
24	14	-1340	
25	14	-1117.5	
26	14	-1077.5	
27	14	-1095	
28	14	-1157.5	and so on.

Sometimes, the data points are measured in an irregular pattern:

6.3	-0.3	81.2
6.3	1.6	80.9
7.1	-4.3	80.9
6.2	-2.5	81.6
8.2	-1.6	81.9
6.1	-0.8	81.4
7.2	-0.3	82.6
5.2	0.4	81.8
6.1	1.0	81.7
5.2	1.6	82.0
5.1	2.2	81.5
6.1	2.8	81.3

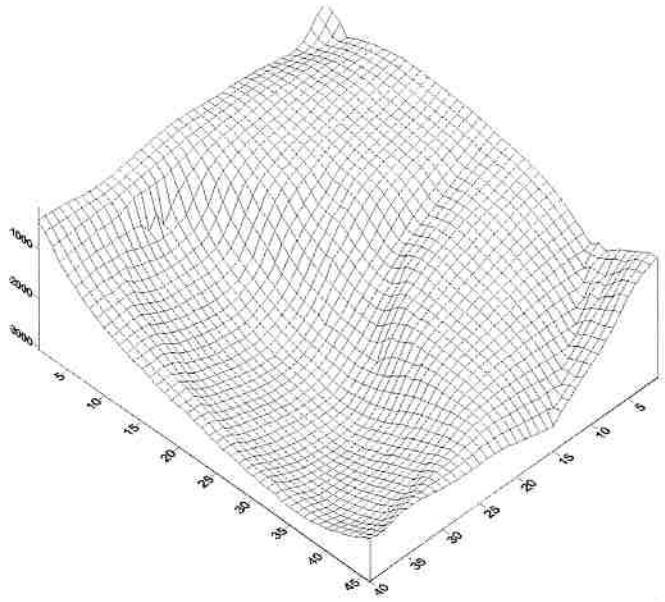

Figure 1: sample digital elevation model on a regular grid, with vertical exaggeration.

DEMs are created by surveyors more often than you realise at first. For a start, it is obvious that all surveys of topography using a total station and reflector create DEMs (ignoring points of detail) even though the surface may later be contoured. But we also should include:

- $ hydrographic surveys, (where again the surface may later be contoured);
- $ photogrammetric mapping by automated software (which invariably creates a DEM before subsequently creating orthoimages and/or contours);
- $ aerial laser scanner surveys; and
- $ terrestrial laser scanner surveys.

Not only are DEMs being created often, a second point to be made about DEMs is that the demand for them is likely to increase. We already see that the mapping organisation sell "digital products", including DEMs, across this state or country, and the client is likely to demand them more and more often because of the suitability for use in computer software for engineering modelling and graphical output for example.

The spacing of DEMs

The concern with this paper is simply whether the spacings chosen for digital elevation models are right. This problem is worst when surveyors are not able to walk the ground and select the surveyed points, as happens with hydrographic, photogrammetric and laser scanner surveys. We have to be concerned about whether the spatial data collection method places these points at the crucial points. In particular, if a terrestrial or aerial laser scanner is used, it might give plenty of points, but they will not necessarily be at the points that a surveyor walking the field would have selected. In other words, what is important about a DEM is that the points do not necessarily appear at the crucial changes of grade that would normally be selected by a surveyor.

Selecting the right spacing for points in a DEM might seem like a trivial problem, but in fact it is quite hard to get to the point where the spacing is completely compatible with the topography. Presumably, the spacing is often selected by intelligent guess work. The point being made in this paper is that it is often hard to justify the selected spacing.

For example, consider a hydrographic survey of a river which is a harbour being used for light shipping, say fishing trawlers and so on, which we could traverse with an echo-sounder. What spacing would we have between the lines? 250 m sounds reasonable - but why? If we can double the spacing to 500 m, we may be halve the cost of the survey, of course.

There is a statistical way of coming up with an answer, even though it must be remembered that it is statistical, and therefore is general and does not ensure that each individual, point is ideal. At least it gives us a way of selecting the point spacing which allows us to justify the selection. The purpose of this paper is to introduce in the simplest way, the idea that something can be done to select the optimum spacing.

A Method for Selecting the DEM Point Spacing

When a DEM is used in practice, the user almost always assumes a straight line between the measured points. Consider four points at the corner of a square in a DEM. We are saying that the user normally adopts a "bilinear" surface between the four corners. A DEM spacing which is too wide then introduces an interpolation error due to the curvature of the true surface. The volume calculated from such a DEM is clearly wrong. We do not want too few points to pick up the curvature; on the other hand, too many points can be wasteful. Presumably therefore we want the situation where the measurement error is compatible with the interpolation error.

Overall, therefore, the basic approach is that the three relevant quantities we need to work and which need to be compatible:

- the spacing;
- the measurement accuracy, and
- the nature of the curvature of the topography.

The most important question here is: how do we describe the surface texture?

We can estimate the required spacing by a simple formula (derived in the Appendix for any interested readers):

$$d = -\sigma^2 / 2s$$

where

- d is the spacing is the required spacing
- σ^2 is the accuracy of the measurements
- most importantly, s is the slope of a certain curve, the covariance function, which describes the surface texture.

The curve slope should be negative, which means that d will be positive.

The real problem with all of this is that the necessary analysis cannot be completed until the area is surveyed because, until then we do not have a way of describing the curvature of the surface. However, it can still be done afterwards, as guidance, and it can be useful for an area which is surveyed regularly, such as a harbour for dredging.

Surface Covariance Functions

A covariance function is a measure of spatial similarity, which is utilised in studies of many physical quantities which are distributed in two-dimensions; see for example Davis [1]. The surfaces of interest are assumed to be defined by a set of measured three-dimensional co-ordinates, (X,Y,Z), where Z is the height coordinate. For any given surface, the sample covariance function at spacing dX, denoted by C, is determined as the mean product of heights at different surface separation distances dX:

$$C_{dX} = (1/N) \Sigma [Z(X).Z(X+dX)]$$

surface heights Z(X) and Z(X+dX) being measured points separated by distance dX, N denoting the number of pairs of points separated by dX, and where the surfaces have been reduced to a mean height of zero. The units of C are the square of the units used for Z.

A covariance function is shown in Figure 2. It is seen that, if N is sufficiently large, the values C can form a surprisingly smooth function. A large value of C occurs when the two surfaces are similar at a given separation distance. In particular, when dX is zero, C has its maximum value. This value, C_0, is called the auto-covariance. As dX increases from zero, the C values normally decrease, because the heights become less correlated with increasing separation, and a covariance function can reaches the value zero at a certain point separation distance, when the surfaces heights

are not correlated to each other. Following a zero covariance the value of C can become negative, remain at zero or become positive again. The last occurs if the surface shows some periodicity. The auto-covariance value and zero-crossing distance together can effectively characterise the surface texture. The slope of the curve also symbolises the rapidity of textural variation within the surface, as it is high for surfaces with little similarity between points - i.e. high curvature.

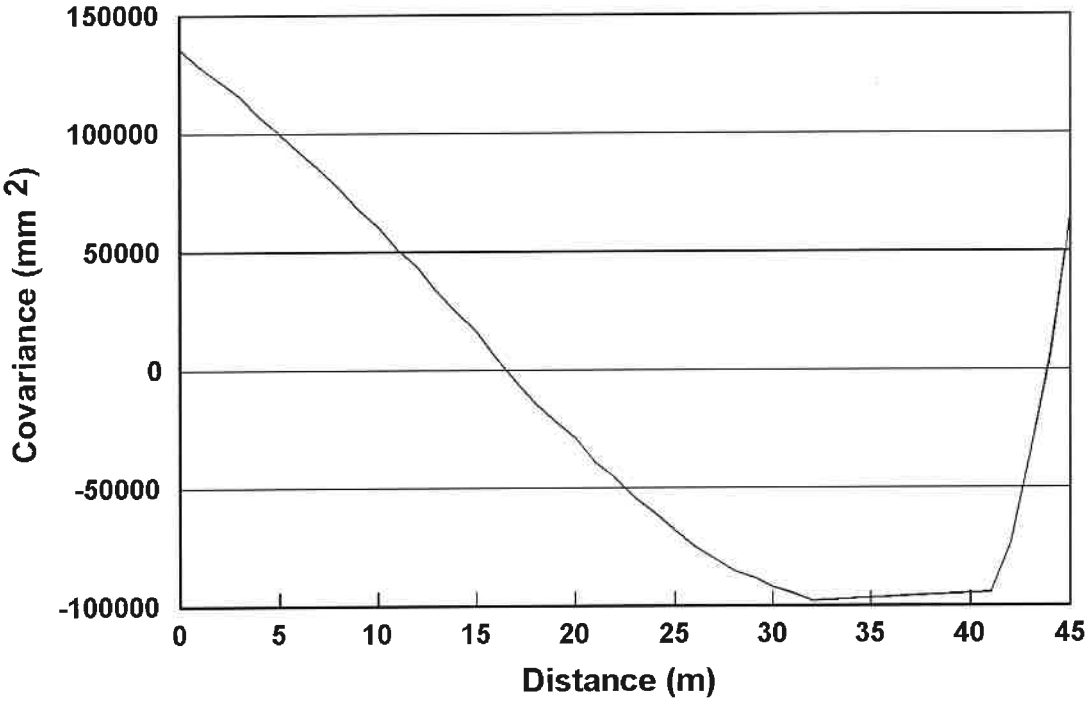

Figure 2: Covariance function for the surface shown in Figure 1

Example

The method is most easily described by a simple example. Suppose our surface is defined by only 20 points, on a regular grid:

	0.9	1.8	2.2	3.3
	1.1	2.1	2.9	3.4
	2.8	3.1	3.0	4.0
	3.1	3.8	4.2	3.3
3.9	4.1	3.1	2.8	

The DEM appears as Figure 3.

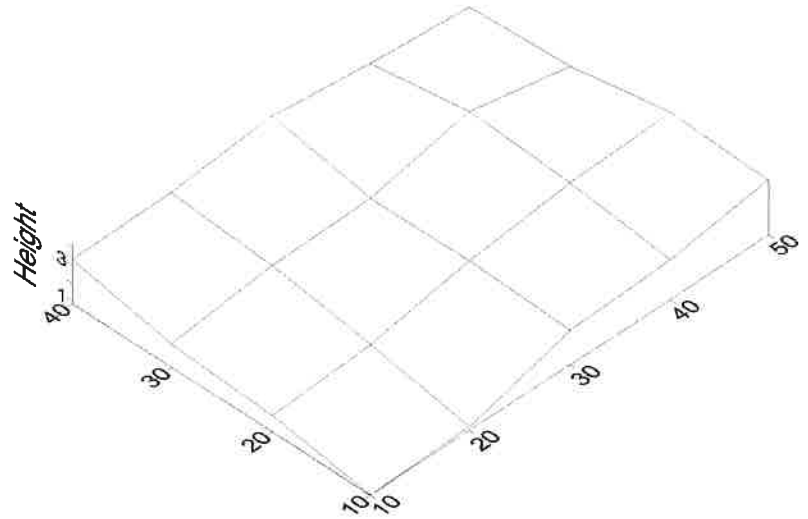

Figure 3: Example DEM with 2:1 vertical exaggeration

The covariance function for this surface is shown in Figure 4.

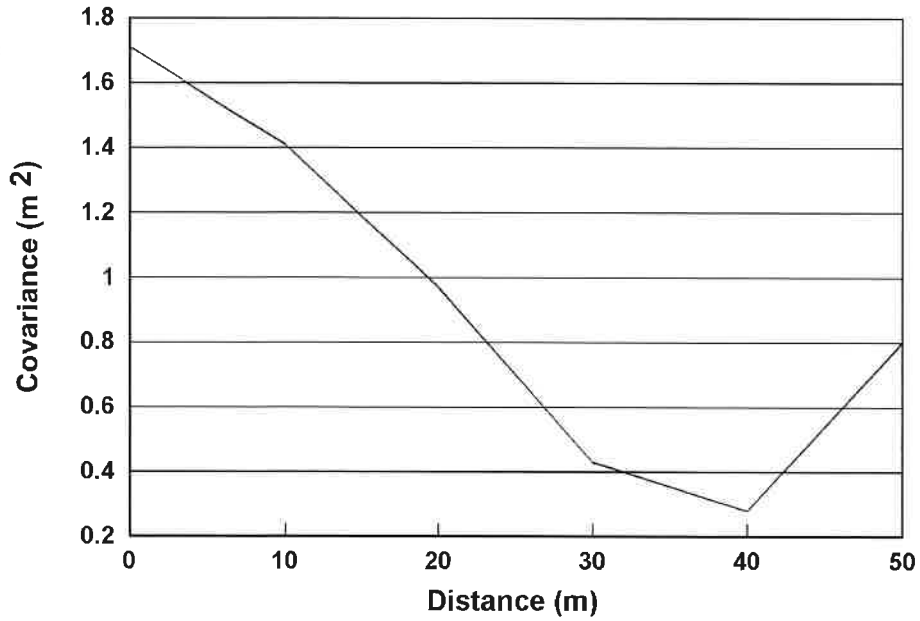

Figure 4: Covariance function for sample surface

The slope is easily seen to be:

$$S = (0.2 - 1.8) / 40$$

$$= -0.04 \quad \text{(metres}^2\text{ / metre)}$$

So we can estimate

$$d = -\sigma^2 / 2s$$

$$= -(0.1)^2 / (2 \times -0.04)$$

$$= 0.125 \quad \text{(metres)}$$

This result suggests that the DEM points should have been taken at a spacing of only 0.1 metre! Our measurement accuracy is way too high for such a wide spacing!

To check: the interpolation error for 0.1 m spacing is given (see Appendix) by:

$$\sigma_{interp}^2 = -0.56\, d\, s$$

$$= -0.56 \times 0.1 \times (-0.04)$$

$$= 0.0022$$

$$\sigma_{interp} = 0.05 \quad \text{(metres)}$$

while the measurement error is given (see Appendix) by

$$\sigma_{mean}^2 = (1/4)\,(0.1)^2 \quad \text{(metres}^2\text{)}$$

$$= 0.0025 \quad \text{(metres}^2\text{)}$$

$$\sigma_{mean} = 0.05 \quad \text{(metres)}$$

If we leave the spacing at 10 m , the interpolation error is:

$$\sigma_{interp}^2 = -0.56\, d\, s$$

$$= -0.56 \times 10 \times (-0.04)$$

$= 0.22$

$\sigma_{interp} = 0.5$ (metres)

A volume calculated with this data is likely to be based on depths which are about 0.5 metres wrong.

One alternative of course is to use a higher accuracy interpolation method.

Conclusion

It is expected that the demand for DEMs will increase, but the writer is concerned that surveyors will find it hard to justify the spacings they select for DEMs. This paper is intended to say that there are ways of selecting the spacing, but they may require some knowledge of the terrain before the selection can be made. If the error caused by the curvature of the ground is to be compatible with the precision of the survey measurements, the recommended spacings will often turn out to be closer than expected.

Reference

Davis J.C., 1986. *Statistics and Data Analysis in Geology*. 2nd Edn. Wiley, New York. Pages 1-646.

Appendix

Relationship between interpolation error and random error

Firstly, the mean-square interpolation error is:

$$\sigma_{interp}^2 = (1/N) \Sigma [Z - Z_{mean})]^2$$

$$= (1/N) \Sigma [Z - \tfrac{1}{4} (Z_1 + Z_2 + Z_3 + Z_4)]^2$$

$$= (1/N) \Sigma [Z^2 + (1/16) (Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2) - \tfrac{1}{2} (ZZ_1 + Z Z_2 + Z Z_3 + Z Z_4) + (1/8) (Z_1Z_2 + Z_1Z_2 + \ldots Z_3Z_4)]$$

$$= (1/N) \Sigma (Z^2) + (1/16) \Sigma (Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2) - \tfrac{1}{2} \Sigma (ZZ_1 + Z Z_2 + Z Z_3 + Z Z_4) + (1/8) \Sigma (Z_1Z_2 + Z_2Z_3 + \ldots Z_3Z_4)$$

$$= C_0 + (1/16) \times 4 \times C_0 - \tfrac{1}{2} \times 4 \times C_{d\sqrt{2}} + (1/8) [4 \times C_d + 2 \times C_{d\sqrt{2}}]$$

using the definition of C

$$= (5/4) C_0 - 2 C_{d/\sqrt{2}} + \tfrac{1}{2} C_d + (1/4) C_{d\sqrt{2}}$$

If the slope of the covariance function is s, then

$$C_{dX} = C_0 - dX s$$

then

$$\sigma_{interp}^2 = - 2 d/\sqrt{2} s + \tfrac{1}{2} d s + (1/4) d\sqrt{2} s$$

because all the C_0 cancel, and so

$$\sigma_{interp}^2 = d s (-2/\sqrt{2} + \tfrac{1}{2} + (1/4) \sqrt{2})$$

$$= d s (-\sqrt{2} + \tfrac{1}{2} + (1/4) \sqrt{2})$$

$$= d s (\tfrac{1}{2} - (3/4) \sqrt{2})$$

$$= -0.56 d s$$

Notice that s is negative, so σ_{interp}^2 will be positive.

Secondly, the mean square error at the central point due to random measurement error is:

$$\sigma_{mean}^2 = (1/4) \sigma^2$$

When these are set to be equal,

$$(1/4) \sigma^2 = -0.56 d s$$

so

$$d = (1/4) \sigma^2 / (-0.56) s$$

or

$$d = (-0.45) \sigma^2 / s$$

or

$$d \quad . -\sigma^2 / 2s$$

Note: As s increases, the slope is greater, and d decreases, as expected.

As σ^2 increases, d increases, also as expected.