GPS HEIGHTING

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ABOUT THE AUTHOR

FRANK SMITH completed his Bachelor of Surveying at the University of NSW in 1967 and became Registered in 1969.

Frank has twenty years experience in control, tunnel and deformation surveys. Was team leader of surveying component of Sydney Water's Ocean Outfalls project. In 1969, he won an Australian Excellence in Surveying Award for document detailing project.

Frank is currently employed as a Project Surveyor, Development Surveys, AWT Survey, with a particular interest in GPS and adjustment theory.

ABSTRACT

This paper reports on the experiences of AWT SURVEY staff with GPS heighting. Because of our close relationship with Sydney Water, establishment of reliable Australian Height Datum heights has been a major component of GPS operations since our purchase of GPS receivers in 1992.

Issues to be raised include...

- Factors affecting GPS heights.
- Convergence, repeatability and accuracy of rapid static, stop go and kinematic methodology.
- Optimal baselines.
- Processing strategies.
- Transformation from WGS84 system to Australian Height Datum.
- Comparison with direct levelling.

1. INTRODUCTION

1.1. Equipment

Three Wild System 200 GPS receivers were purchased in 1992, capable of operation in Static, Rapid Static, Stop Go and Kinematic modes. Although Rapid Static is used for the bulk of heighting projects, heights derived by each method are analysed.

Geocentric baselines and baseline correlations are output by Leica SKI software supplied with the GPS receivers. Resolved baselines are exported from SKI in an ASCII file format for subsequent processing and analysis.

Import and adjustment of baselines, error analysis and transformation into the AHD heighting and local horizontal system are performed by the COMPNET three dimensional adjustment program.

1.2. Experience

The receivers were purchased in 1992 principally to monitor three dimensional tectonic and local stability in the Cataract Dam region stretching from the coast near Wollongong to Cataract Dam. They have subsequently been used on numerous additional projects for which height establishment is the major component, including..

- Work as executed surveys of sewerage and stormwater assets.
- Heights on pumping stations throughout the North Western part of Sydney.
- Flood level investigations in the Upper Nepean River system.

- A further three dimensional tectonic survey within the Nepean Dam catchment area.
- Measurement of subsidence caused by longwall coal mining passing under Cataract River.
- Profiles of routes of major water mains.

1.3. Test area

While this paper makes use of information derived from all of the above projects plus specific tests, data from a flood study survey in the Upper Nepean catchment from Wallacia and Warragamba to Cobbitty, a north-south distance of 18km (refer Figure 1) is prominent.

This area provides an interesting study of several methods of establishing AHD control by GPS. It comprises terrain rising from the Nepean River (40 - 50m) to the foothills of the Blue Mountains (Warragamba 180m), a variation that may influence the shape of the geoidal surface. The area around Bents Basin is heavily vertically indented with the basin itself a deep body of water, again having possible repercussions for the shape of the geoid. Finally, available second order level control, while adequate in quantity is unevenly distributed, possibly exposing weaknesses in any purely geometric AHD solution.

2. ERROR SOURCES

Numerous papers have identified errors resulting from GPS biases and transformational uncertainties. It is therefore proposed only to summarise those having an effect upon a baseline (differential) solution.

Three general classes of error are identifiable, those caused by...

- The GPS measurement process.
- Conversion of GPS height differences to the AHD.
- Errors in existing AHD vertical control.

2.1. GPS errors

Biases having an impact upon heights derived from a Rapid Static baseline solution include...

- Incorrect ambiguity resolution.
- Mis-modelling of the differential tropospheric correction.
- Relative ionospheric delay errors with single frequency receivers.
- Assumed station position.
- Multipath.

- Tripod heights
- Antenna or sensor phase centre variation

Simulations (Holloway, 1988) suggest that a failure to completely resolve such biases results in a height error of some 4ppm of the baseline for a Static solution with a single frequency receiver - 40mm for a 10km line.

The short span of Rapid Static observations, often of the order of five minutes, increases the possibility of significant multipath errors and also failure to correctly resolve ambiguities, an addition to the error budget of the simulation. Origin (reference station position) errors, however, assumed to be some 1ppm, are less significant as transformations from AMG to WGS84 are, in NSW, accurate to better than 10m (Kinlyside 1992). Refer to 4.1.4. for Rapid Static investigations.

Note that all such effects may be systematic for a single occupation, neither the baseline solution nor subsequent least squares adjustment revealing possibly significant errors.

2.2. AHD conversion

For processed GPS baselines, the height difference refers to the WGS84 ellipsoid. Generally, to be of use for engineering and development applications, such heights must be transformed to the Australian Height Datum, either by means of a seven parameter transformation (the geometric method), or by the application of gravimetrically determined WGS84 geoid-spheroid separations.

The geometric method rotates the GPS vectors onto a minimum of three (or more for redundancy) vertical control stations, establishing an ellipsoid approximating the slope and position of the geoidal surface defined by the control stations. The latter should be SCIMS AHD values of Third or preferably higher order, located throughout a surveyed area to best represent the geoid at places where height determination is required. As the geometric method represents an "average" AHD surface, geoid undulations may not be adequately modelled. In particular, in areas of major height or geologic variation significant heighting errors may be introduced. Generally, the reliability of the geometric method may also be expected to decrease with increasing area of survey - the maximum area should be assessed from the nature of the terrain, fit of GPS vectors on vertical control stations and the regional stability of the WGS84 to AHD vertical rotational parameters produced by the network adjustment.

A refinement of the geometric method introduces a contouring algorithm to smooth residuals at control stations throughout a network. Errors associated with the GPS survey may be accommodated by comparing fixed minus free heights at a control station and deducting an estimate of the error of GPS fixation based upon the standard deviation of the free height. While contouring minimises the effects of geoid undulations at or close to control stations, it may well not model rapid change in the geoid remote from these.

Network adjustment programs rotate GPS vectors into the AHD system when the user introduces AHD vertical constraint to an adjustment; COMPNET specifically reports on the geoidal slope in a northerly and easterly direction. Optionally such programs also carry out post-adjustment contouring.

Within Australia, precise gravimetrically determined AHD heights may be determined from N (geoid-WGS84 spheroid) values derived from OSU91 global geopotential coefficients and 1980 Australian Gravity data base information. Such N values are computed by AUSLIG Geodesy using software developed by Dr A.H.W. Kearsley of UNSW and are claimed to have an absolute accuracy of better than 1m and relative accuracy of 2 to 5ppm of the GPS baseline length (Kearsley 1988).

AUSGEOID93 is a 10' by 10' grid of such precise N determinations covering the Australian region. Using AUSLIG program WINTER, interpolation of points within a grid is accomplished to a claimed accuracy within a few centimetres of an equivalent precise N determination.

COMPNET imports either precise or interpolated gravimetric N values and, forcing geoidal rotational parameters to zero, produces AHD heights in a constrained or free net adjustment .

2.3. Vertical control

Vertical control errors relate to the accuracy of the original spirit levelling, indicated by the SCIMS vertical order, and mark subsidence since survey. Such errors may in some measure be monitored by observing, if sufficient control exists, GPS vectors to more than one control station in the same local area.

3. METHODOLOGY

3.1. Field

Project accuracy requirements and economic constraints, together with satellite signal strength and geometry at individual stations dictate the duration and sequence of site occupation. Additional constraints may apply; for example, surveys of CLASS B or higher - refer ICSM Publication SP1 - require that sites be reoccupied.

AWT SURVEY staff generally adopt a two reference station, one rover scenario, locating the former in secure locations, not necessarily existing control stations, with a clear window to the sky. One man operation is the norm.

The Leica Rapid Static observational method is used, observation times generally satisfying Leica guidelines for a given reference - rover baseline. Again consistent with guidlines, line lengths are usually less than 5km and rarely greater than 10km.

For baselines less than 5km, an occupation time of 5 to 8 minutes is adequate; additional is allowed if the satellite constellation is weak as indicated by a large Geometric Dilution of Precision (GDOP) or if the signal to noise ratios for any satellites are low.

To minimise phase centre errors, all sensors are oriented in a common direction. Care is taken when measuring antenna height as, unless the station is reoccupied, such errors will not be apparent in subsequent baseline processing and network adjustment; a useful tool when high height precision is paramount is a quick set, constant height tripod, which largely eliminates antenna height reading errors.

In addition to station, height, satellite numbers and GDOP details, note is made of low elevation, weak and otherwise suspect satellites, in particular those at 180 degrees to flat, reflective surfaces. Such may, if necessary, be disabled during processing.

It should be mentioned that, unless the survey is repetitive (for example tectonic or stability measurements), the short duration of station occupation and thus the large numbers surveyed per day usually precludes effective reconnaissance and obstruction identification . It is more economic to "try ones luck" and return later if unsuccessful, particularly as the generally sparse nature of Australian tree cover often creates surprisingly little restriction to a satellite signal.

Multiple occupation at a different sidereal time is mandatory for high precision surveys, tectonic and control densification for example, and is also routinely adopted if initial occupation and subsequent processing gives cause to doubt the reliability of a baseline solution. Such cases include observations to a minimum of four satellites, poor GDOP and weak or low elevation satellites; experience suggests that if the constellation is strong and unobstructed, all reduced baseline components including height are satisfactory.

It should be noted that while two reference receivers give the appearance of redundancy, the contemporaneous nature of both baselines and a common data set at the rover may at least partially hide systematic errors, particularly in the height component. The author has experienced a case with this configuration where, with four satellites and disturbed signal, a height error of 0.2m was not revealed by SKI baseline processing or a subsequent network adjustment. Satellite behaviour should be carefully monitored during occupation.

Preferably second order vertical and horizontal control stations are identified and occupied in the same campaign and from the same reference stations as unknown points. Clearly, to minimise (or identify) the effect of geoid undulations, control should be as close as possible to new stations, particularly if the geometric method is to be adopted to establish final AHD heights.

3.2 Processing

Baseline reductions occur in the Leica SKI program. Generally the software is reasonably intuitive provided the user has a grasp of GPS processes. Defaults facilitate largely automated processing of baselines from Rapid Static, Stop Go and Kinematic data.

To satisfy Leica's claimed (horizontal) static accuracy of 5mm + 1ppm, WGS84 geographic coordinates of the defined origin station of a GPS network should be known to an accuracy of 10m. Applying the AGD66 to NSW GPSNET transformation developed by the LIC to a network station with known SCIMS ISG/AMG horizontal coordinates and AHD heights satisfies this criterion. An alternative but less satisfactory solution in regions where control is sparse or transformation parameters unreliable is to process an extended period of GPS point positions by means of the SKI "Single point processing mode". The WGS84 geographic coordinates for the origin station are stored in the database; baseline computation then begins from this station and radiates throughout the network, ensuring that all stored stations are related and in the WGS84 system.

If no anomalies have been identified during observation, baseline processing is usually successful and the results reliable. If SKI, however, fails to resolve ambiguities, a number of options are available, in increasing order of risk...

- Reoccupy station
- Disable suspect satellites
- If the time of occupation is sufficient, window a segment of the complete observation to eliminate suspect data.
- Increase the RMS - the a priori threshold of phase noise.

Station reoccupation is a preferred option for high precision heighting and may be necessary even after application of the alternative options. Disabling satellites identified in the field or after residual processing as weak (for example L2 failure or low SN, identified as less than 38 for L1 and 35 for L2) or of low elevation (20 degrees or less) is reasonably secure provided baseline precision, GDOP (a maximum of 8 and lower than 5 for high precision) and satellite numbers (5 or more should remain) are adequate. Windowing, while often effective, may lead to unexpected baseline, usually height, errors with disturbed signals. There is no guarantee that ambiguities, if resolved, are correct if the RMS is increased from the default 10mm.

SKI baselines are then imported into the three dimensional network adjustment program, COMPNET, which converts geocentric baseline vectors to a topocentric system (E, N, H) before adjustment. An unconstrained adjustment (one station fixed) aids identification, with the cautions expressed previously, of height and coordinate errors. The final constrained adjustment, holding as fully fixed a minimum of two horizontal points and three vertical, generates AMG/ISG coordinates and AHD heights.

Such geometrically determined heights may optionally be contoured to existing control (refer 2.2.). Should the user have access to AUSLIG WINTER, WGS84 geographics may be exported from COMPNET, N values interpolated in WINTER and then re-imported into the adjustment program. Alternatively, for highest vertical precision, the WGS84 geographics may be rigorously processed by AUSLIG and later directly imported into COMPNET. Two estimates of AHD height are then feasible, one contoured geometric and the other using gravimetrically determined N.

4. HEIGHTING TESTS

4.1 GPS errors

4.1.1. Ionospheric and tropospheric effect

For observing sessions exceeding 45 minutes, SKI will optionally compute an ionospheric model. Refer to Table 1 for a height comparison on a 13.6km line, Bendeela to Tallowa, processed twice in different windows on 9th December 1994, observing sessions each about 1.5 hours.

COMPUTED IONOSPHERE vs STANDARD MODEL			
COMPUTE	STANDARD	DIFFERENCE MM	PPM
254.622	254.618	-4	0.3
254.605	254.609	+4	0.3

Table 1

The ionosphere free and standard model differ by only 0.3ppm. The difference between both vectors by the standard model (9mm or 0.7ppm) is in fact less than is in evidence when the ionosphere is computed (17mm or 1.2ppm).

It is usual for commercial GPS software to model the tropospheric correction. Leica give the user the option of using Hopfield (default), Modified Hopfield, Saastamoinen or none, the latter being recommended for investigative purposes only. Table 2 indicates baseline differences on the same 13.6km line as Table 1.
While No Model is significantly different, those recommended are similar.

Significant unmodelled errors remain (approx 1ppm), possibly attributable to relative tropospheric and ionospheric influences.

ALTERNATIVE TROPOSPHERIC CORRECTIONS		
HOPFIELD	SAASTAMOINEN	NO MODEL
254.605	254.606	254.404

Table 2

4.1.2. Ambiguity resolution

Rapid Static processing makes use of a least squares search technique to establish the combination of whole wave length ambiguities for all observed satellites resulting in a best fit solution. Occasionally the solution fails, as evidenced by Table 3, representing Rapid Static solutions at one minute intervals to 16 minutes. The baseline is from Brownlow Hill to Werombi (7.5km), date 17th October 1994. Five satellites were observed, one, SV15, having an elevation of 17 degrees. WGS84 height difference "truth" is 103.818, standard deviation 3mm.

	ALL SATELLITES		SV15 DISABLED	
INTERVAL	DELTA HEIGHT	ERROR MM	DELTA HEIGHT	ERROR MM
2	102.990	-828	FAIL	NA
3	102.996	-822	FAIL	NA
4	103.001	-817	103.776	-42
5	103.003	-815	103.774	-44
6	103.004	-814	103.772	-46
7	103.006	-812	103.768	-50
8	FAIL	NA	103.767	-51
9	FAIL	NA	103.767	-51
10	103.806	-12	103.769	-49
11	103.807	-11	103.772	-46
12	103.807	-11	103.774	-44
13	103.807	-11	103.774	-44
14	103.807	-11	103.776	-42
15	103.806	-12	103.777	-41
16	103.806	-12	103.779	-39

Third order level tolerance 33mm

Table 3

For very short observation spans, less than the minimum 8 minutes flagged by SKI as necessary for a solution at this baseline length, gross height errors (-820mm approx) are apparent. Clearly ambiguities, while reported as resolved, are incorrect for the satellite combination.
With SV15 disabled and satellites reduced to four, the gross error is eliminated.

4.1.3. Station position

As discussed previously, provided the user employs an AMG to WGS84 transformation appropriate to the region, WGS84 geographic coordinates are accurate to within 10m, sufficient to meet Leica's accuracy specifications. Should, however, a single point position be adopted as the origin, station position errors of as much as 200m are possible. Heights differences for two lines, 4.6km and 13.6km, have been processed with station coordinates in the SKI database in total error by 200m (45 degree direction), height by 100m and additionally with this same error carried into the network adjustment. The results appear in Table 4.

PRECISE WGS84 V POINT POSITION					
LINE	PRECISE	SKI	DIFF	LEAST SQ	DIFF
4.6km	32.233	32.243	+10	32.348	+115
13.6km	254.605	254.491	-114	254.308	-297

Table 4

Height errors potentially resulting from use of point positions for processing and subsequent least squares adjustment are well demonstrated. Errors of up to 8ppm are produced from SKI, increasing to 25ppm in the least squares component. As horizontal station errors represent a datum rotation, the vertical rotations of a constrained geometric adjustment will to some extent compensate, but at the cost of meaningless rotations. Heights in an unconstrained adjustment using gravimetrically determined N will be in error and those produced when the latter is constrained will generate excessive residuals at constrained stations.

Users should ensure that, for the origin station of a GPS network, transformed WGS84 geographic coordinates and heights, not a point position, is stored in the GPS software database before baseline processing commences.

If using COMPNET least squares software for an ISG/AHD adjustment, the fixed free station net should be a SCIMS value, order 4 or better, and WGS84 global geographic and height corrections must be capable of converting the fixed coordinate to a WGS84 equivalent with an accuracy of better than 10m - 0.3" longitude and latitude.

4.1.4. Rapid Static

Tests have been carried out on lines of 4.6km and 7.5km to assess such issues as rapidity of convergence to a stable height difference, effect of baseline length upon measurement and dominant factors in a height determination. Refer to Tables 3, 5 and 6. Tables 5 and 6 refer to line Brownlow to Mt Hunter, a distance of 4.6km, observed on 26th and 27th October 1994; "true" WGS84 height difference for this baseline is 32.238, standard deviation 1mm. Observation intervals are in minutes.

	ALL SATELLITES		SV15 DISABLED	
INTERVAL	DELTA HEIGHT	ERROR	DELTA HEIGHT	ERROR
2	32.234	-4	32.234	-4
3	32.235	-3	32.235	-3
4	32.236	-2	32.236	-2
5	32.237	-1	32.239	+1
6	32.236	-2	32.238	0
7	32.237	-1	32.238	0
8	32.240	+2	32.241	+3
9	32.242	+4	32.243	+5
10	NIL	NA	32.245	+7
11	32.247	+9	32.246	+8
12	32.248	+10	32.247	+9
13	32.252	+14	32.249	+11
14	32.253	+15	32.251	+13
15	32.254	+16	32.251	+13
	¦Average¦	6.3	¦Average¦	5.5

Third order level tolerance 26mm

Table 5

Despite SKI reporting insufficient data for the first four minutes, Table 5 indicates that the solution with 6 satellites locks onto the correct height difference in two. Low elevation SV15 (16 degrees) caused a solution failure when processing 10 minutes of data. Disabling SV15 does not significantly influence the navigation precision, causing the GDOP to increase from 2.8 to 3.5; the solution is marginally improved. In both cases the solution deteriorates with increased occupation time.

	ALL SATELLITES		SV16 DISABLED	
INTERVAL	DELTA HEIGHT	ERROR	DELTA HEIGHT	ERROR
2	32.265	27	32.234	-4
3	32.271	33	32.244	+6
4	32.274	36	32.253	+15
5	32.271	33	32.253	+15
6	32.270	32	32.255	+17
7	32.267	29	32.256	+18
8	32.264	26	32.253	+15
9	32.265	27	32.253	+15
10	32.264	26	32.253	+15
11	32.264	26	32.252	+14
12	32.263	25	32.250	+12
13	32.264	26	32.248	+10
14	32.263	25	32.247	+9
15	32.263	25	32.247	+9
	¦Average¦	28	¦Average¦	12

Third order level tolerance 26mm

Table 6

Refer Table 6, again SKI indicates a minimum of four minutes for an adequate solution, but the result is satisfactory, if inferior to that of Table 5, in two with 6 satellites. The comparatively poor result is partly due to the effect of SV16 (elevation 21 degrees) which does not, after inspecting residuals, appear to be detrimental to the result in the full solution but has a significant influence upon the GDOP (2.7 enabled, 5.5 disabled).

Table 3 (refer earlier) analyses observations over a 7.5km line. Here SV15 (17 degrees elevation) first causes a gross error. When SV15 is disabled, the early gross error is eliminated, GDOP increases from 4 to 7, satellites are reduced to 4 but the result at realistic occupation intervals is less accurate than that with it enabled.

Conclusions and recommendations that may be drawn from the Rapid Static tests are...

- There is no advantage in occupying more than the minimum recommended time - results may actually diverge with additional.

- Low elevation satellites, particularly those having a major effect upon navigation precision (GDOP), may significantly affect a height solution. If suspect disable, note GDOP change and solution variation. The superior solution, provided more than five satellites remain and GDOP is 5 or less, is more likely to be the disabled case.

- If disabling low elevation or suspect satellites, ensure five or more remain.

- The possibility of gross error increases with baseline length - for high precision height measurement restrict baseline length to 5km.

- If in doubt, reoccupy.

- Ensure that redundant vectors or other measurements exist for each unconstrained point and carry out a rigorous free net least squares adjustment of the full network. Reject vectors with excessive residuals and be aware of possible systematic errors.

4.1.5. Stop Go and Kinematic

Both procedures are similar, ambiguities being resolved at the commencement of a chain of points either by occupation of a known station or a Rapid Static occupation, and carried throughout the survey by maintenance of lock onto a minimum of four satellites. Subsequent station occupation times for Stop Go are two to four epochs, a discrete three dimensional position being established for each point. The Kinematic procedure generates vectors at fixed time intervals, with the added, and useful, Leica feature of time tagging, allowing individual points to be identified in the field and extracted from a Kinematic chain.

Leica claim an achievable height accuracy of 2 to 6cm + 1ppm for both Stop Go and Kinematic. To test this claim Tables 7 and 8 respectively detail Stop Go and Kinematic measurements at Prospect Reservoir, date 22nd December 1994. Stations occupied are part of the Prospect deformation survey; known heights are derived from this. Baseline length is 5.1km, initialisation occupation times are Leica recommended < 5km 100%, Stop Go interval is 4 10sec epochs and Kinematic time tag method is Next Epoch.

Height on the reference station is derived from the known height of point 35 and the first initialisation vector.

The results for both Stop Go and Kinematic are within Leica minimum tolerances. Means of forward and reverse measurements are all within 20mm of truth and generally better than 10mm.

STOP GO V KNOWN HEIGHTS				
ID	HEIGHT	GPS HEIGHT	DIFF MM	MODE
35	63.276	63.276	0	INIT
35	63.276	63.297	21	SGO
35	63.276	63.288	12	SGO
35	63.276	63.296	20	SGO
35	63.276	63.297	21	SGO
35	63.276	63.296	20	SGO
36	63.217	63.224	7	SGO
37	63.132	63.130	-2	SGO
38	63.108	63.118	10	SGO
39	62.921	62.927	8	SGO
40	62.835	62.837	2	SGO
41	62.705	62.702	-3	SGO
41	62.705	62.711	6	CLOSE
41	62.705	62.723	18	INIT
40	62.835	62.831	-4	SGO
39	62.921	62.926	5	SGO
38	63.108	63.110	2	SGO
37	63.132	63.132	0	SGO
36	63.217	63.211	-6	SGO
35	63.276	63.288	12	SGO

Table 7

KINEMATIC V KNOWN HEIGHT				
ID	HEIGHT	GPS HEIGHT	DIFF MM	MODE
41	62.705	62.703	-2	INIT
40	62.835	62.842	7	KIN
39	62.921	62.922	1	KIN
38	63.108	63.120	12	KIN
37	63.132	63.135	3	KIN
36	63.217	63.217	0	KIN
35	63.276	63.272	-4	KIN
35	63.276	63.275	-1	KIN
35	63.276	63.267	-9	KIN
35	63.276	63.273	-3	KIN
35	63.276	63.276	0	INIT
36	63.217	63.223	6	KIN
37	63.132	63.119	-13	KIN
38	63.108	63.104	-4	KIN
39	62.921	62.931	10	KIN
40	62.835	62.835	0	KIN
41	62.705	62.707	2	KIN
41	62.705	62.708	3	KIN
41	62.705	62.711	6	KIN

Table 8

4.2. AHD conversion

As discussed previously, AHD heights may be derived from GPS WGS84 vectors by geometric or gravimetric means. The test area, refer Figure 1 and section 1.3., provides an opportunity to compare and assess AHD heights derived geometrically (with contours), by WINTER interpolation and using precise N computed by AUSLIG Geodesy. Data is adjusted by COMPNET software, the geometric version of the output, project name WB151, having been transmitted to C&LM, Control Survey Branch. The survey comprises 40 unconstrained stations, 7 fully constrained and 12 constrained vertically.

4.2.1. WINTER

As the WINTER program derives N for a point by bicubic interpolation from the nearest elements on a 10' by 10' grid of precisely determined N values, direct comparison of N by WINTER and directly computed (precise) N in the test area represents a measure of the reliability of WINTER interpolation. Refer to Figure 2, a contour plot of differences in millimetres, which indicates that, over distances of less than 8km, ΔN by both methods may vary by up to 100mm, a figure at variance with the claimed few centimetres. AHD heights derived by applying N by WINTER to GPS vectors may, in the test area, contain similar errors from this source alone .

Table 9 is a comparison from WB151 of most unconstrained station heights by the contoured geometric method, precise gravimetric N and WINTER interpolation. The absolute average of the differences (Precise - Geometric), $\Delta 1$, and (WINTER - Precise), $\Delta 3$, suggest that he geometric solution is superior to WINTER in the test area. WINTER models undulations (maximum coordinate differences) less well than contoured geometric.

Provided precisions, number of vectors etc are common, change of variance factor is a measure of fit of an adjustment on vertical control. For the geometric solution this is 0.39, precise N - 0.52 and WINTER - 0.62. While unconstrained rotational parameters in the geometric case reduce the variance factor compared to that where N values exist (forced to zero), N by WINTER clearly does not fit as well a precise N.

		GEOMETRIC V	PRECISE N V		WINTER		
ISG ZONE 56/1			AHD		HEIGHTS		
E	N	GEOM	PRECISE	Δ1	WINTER	Δ2	Δ3
268997	1236927	61.728	61.756	+28	61.766	+38	+10
266379	1243650	49.479	49.515	+36	49.470	-9	-45
266198	1244552	52.165	52.198	+33	52.150	-15	-48
267068	1240583	62.315	62.335	+20	62.330	+15	-5
264071	1251909	57.366	57.372	+6	57.344	-22	-28
266216	1243116	48.239	48.278	+39	48.234	-5	-45
265500	1251616	23.079	23.077	-2	23.062	-17	-15
265578	1251767	51.043	51.034	-9	51.027	-16	-7
265694	1251362	49.298	49.292	-7	49.284	-14	-7
266142	1250217	39.729	39.719	-10	39.719	-10	0
266022	1249610	42.245	42.246	+1	42.237	-8	-9
265708	1248443	47.737	47.737	0	47.732	-5	-5
265948	1246884	48.021	48.023	+2	48.009	-12	-14
266157	1245125	53.517	53.547	+30	53.499	-18	-48
266158	1245380	52.497	52.526	+29	52.490	-7	-36
266154	1245161	53.372	53.402	+30	53.354	-18	-48
266160	1244251	47.356	47.389	+33	47.355	-1	-34
266096	1244176	44.360	44.395	+35	44.360	0	-35
265931	1242509	69.954	69.994	+40	69.953	-1	-41
264683	1240966	46.714	46.742	+28	46.726	+12	-14
265727	1240079	57.867	57.878	+11	57.887	+20	+9
266096	1236869	77.985	77.991	+6	78.031	+46	+40
267232	1238269	58.423	58.446	+23	58.451	+28	+5
268673	1237053	63.197	63.230	+33	63.235	+38	+5
269349	1237552	60.993	61.013	+20	61.040	+47	+17
268597	1235501	73.785	73.811	+26	73.824	+39	+13
267710	1235361	61.468	61.480	+12	61.511	+43	+31
268187	1233428	74.442	74.460	+18	74.448	+6	-12
264682	1244611	133.443	133.454	+11	133.434	-7	-18
263418	1249739	114.434	114.409	-25	114.423	-11	+14
264731	1249652	168.550	168.536	-14	168.535	-15	-1
264137	1239733	177.990	178.015	+25	178.007	+17	-8
262706	1248310	180.942	180.933	-9	180.935	-7	+2
265827	1247212	53.179	53.181	+2	53.165	-14	-16
266134	1245757	49.465	49.488	+23	49.457	-8	-33
264450	125158	27.923	27.914	-9	27.905	-18	-9
265104	1242602	110.912	110.953	+41	110.911	-1	-42
266581	1243889	46.264	46.302	+38	46.254	-10	-48
			¦Av¦	20		¦Av¦	21.5
			¦Max¦	41		¦Max¦	48

Table 9

In areas where vertical control is limited, WINTER is an effective means of establishing AHD heights from GPS vectors. As this program uses rigorous AUSGEOID93 N as a base, interpolation errors are also likely to be relatively less significant on longer lines. For smaller areas with adequate vertical control it offers no advantages over a geometric solution.

4.2.2. Precise N

A least squares adjustment in which precise N values have been applied to GPS vectors should fit existing AHD control with no distortions - unconstrained vertical rotations should statistically be zero. Table 10, computed using a non-standard adaption of COMPNET with unconstrained vertical rotations and N existing, indicates that, at 99% level, significant rotational distortions are present.

VERTICAL ROTATIONS WITH PRECISE N			
East Rotation	0.70"	Std Dev	0.12"
North Rotation	-1.37"	Std Dev	0.22"

Table 10

The resultant rotation, 1.53" or 7.5 ppm, exceeds the claimed Precise N relative accuracy of 2 to 5ppm.

4.2.3. Geometric method

Figure 3 is a contour plot in millimetres of the differences between heights extracted from the least squares adjustment using precise N and the geometric method. It may therefore be considered a measure of geoid undulations relative to the best fit ellipsoid of the geometric method. These reach a maximum of approximately 40mm in the area close to Bents Basin. This region is topographically varied, combining deep water and rapidly varying elevation, and is also remote from adopted vertical constraint.

PM54320, a 4th Order vertical SCIMS station, is situated in the Bents Basin reserve. Unconstrained in the adjustment, it provides, consistent with its relatively low accuracy, a basis of comparison of the three methods of AHD conversion as indicated on Table 11.

PM54320 COMPARISON						
SCIMS	GEOMETRIC	Δ	PRECISE N	Δ	WINTER	Δ
49.54	49.479	-61	49.515	-25	49.470	-70

Table 11

Two facts are apparent: precise N best models the geoid at this location and WINTER does so less well than the geometric method.

Some 2.4km due north of PM54320, at the edge of the area defined by Figure 3 as containing significant undulations, the first vertical constraint occurs - SSM35427, 2nd Order SCIMS. Table 12, indicating vertical residuals on this station, further reinforces the relative merit of the three methods.

SSM35427 RESIDUALS		
GEOMETRIC	PRECISE N	WINTER
+20	+2	+38

Table 12

4.2.4. Summary

Using the contoured geometric method alone results in height errors that are related to....

- Size of project area.
- Density of vertical control.
- Proximity of control to unconstrained heights.
- Irregularity of terrain.

In the test area, all such factors are less than ideal. Despite this, differences to an adjustment using precise N averages -15.5mm, with maximum errors of some -40mm. For many purposes this is adequate.

Remembering that such errors are additional to those of the GPS measurement, for high precision (third order?) applications, precise N should be computed by AUSLIG and included in the network adjustment. This solution may then be compared to an initial geometric adjustment. Agreement gives confidence in a result, differences may be resolved by additional vertical constraint.

When extrapolating from existing control or where such is sparse or of poor quality (lower than 3rd Order), the geometric method becomes inappropriate. Constrained adjustments then should always include gravimetrically determined N (preferably precise) with vertical rotational parameters set to zero.

5. TOTAL ERROR BUDGET

Provided major sources such as ambiguity and station position errors are eliminated, a realistic estimate of Rapid Static baseline errors to be used in the least squares process is....

Horizontal	10mm + 2ppm
Vertical	15mm + 2ppm

Using such apriori baseline standard deviations, free net adjustment variance factors of 0.20 to 0.60 are the norm, low because unmodelled systematic errors are present.

A conservative approach is therefore to consider the absolute heighting precision from a network adjustment as the vertical standard deviation, unscaled by the variance factor unless this is greater than unity.

The vertical CLASS of stations in the network may be assessed by comparing the unscaled relative vertical standard deviations derived from a free net adjustment to the formula....

$$r = c\sqrt{d}$$

where r is the maximum allowable error in mm, c an empirically derived factor for each CLASS and d is the distance between stations to which the relative standard deviation applies; c for CLASS A, B and C respectively is 4, 8 and 12.

The vertical ORDER of the survey refers to the final adjustment constrained to existing vertical control. It is a function of the CLASS and the ORDER of such existing control. Refer to ICSM publication SP1, "Standards and Practices for Control Surveys".

Significantly, the above relative and absolute precisions do not include errors arising from the transformation to the AHD - relative N errors if gravimetric methods are employed, estimates of geoid undulations for the geometric method. A realistic total error budget would combine estimates for such errors (at 1 standard deviation) with absolute or relative precisions from the least squares output. If using precise N, perhaps 5ppm from the nearest vertical constraint is appropriate.

For the geometric method, undulation is terrain etc dependent (see 4.2.4.); in the test area a suitable figure would be 16 - 20mm. Experience in a particular region, assessing such factors as fit on vertical control (a specific module in COMPNET), direct levelling checks (refer 6.) and the stability of the vertical rotational parameters in successive surveys, will allow a reasonable estimate to be developed.

6. DIRECT HEIGHT COMPARISONS

Twenty seven State Survey Marks in the region from Mt Victoria to Parramatta have been heighted both by GPS and 3rd Order levelling (refer Table 13, differences by Sydney suburb). GPS observation mode is Rapid Static, usually a single occupation with double reference stations. The geometric method is employed to produce AHD heights.

Particularly in the Blue Mountains, the results are obtained under difficult conditions and thus probably overstate the errors expected. Typical problems encountered were heavy tree cover with consequent broken and/or low satellite signals (usually L2), and proximity to structures.

SUBURB	GPS	LEVELLING	DIFF
CAMELLIA	4.441	4.413	-28
NORTHMEAD	14.189	14.173	-16
RICHMOND	18.556	18.545	-11
ST MARYS	24.072	24.041	-31
PENRITH	24.419	24.417	-2
PENRITH	25.410	25.411	+1
PENRITH	27.272	27.281	+9
PENRITH	24.980	24.987	+7
DOONSIDE	46.437	46.472	+35
LAPSTONE	106.760	106.776	+16
GLENBROOK	145.690	145.701	+11
GLENBROOK	152.640	152.644	+4
GLENBROOK	158.388	158.406	+18
BLAXLAND	207.671	207.680	+9
WARIMOO	245.130	245.165	+35
VALLEY HEIGHTS	299.289	299.354	+65
SPRINGWOOD	334.046	334.040	-6
SPRINGWOOD	410.555	410.539	-16
HAZELBROOK	610.390	610.430	+40
HAZELBROOK	651.510	651.450	-60
BULLABURRA	738.164	738.160	-4
WENTWORTH FALLS	845.950	845.920	-30
WENTWORTH FALLS	904.890	904.900	+10
NORTH KATOOMBA	618.940	618.930	-10
BLACKHEATH	1004.232	1004.212	-20
BLACKHEATH	1050.970	1050.943	-29
MT VICTORIA	1045.442	1045.448	+5
			¦AV¦ 19
			STD DEV 3

Table 13

In addition, the geometric method may not model all geoid undulations, which, in mountain terrain, are possibly significant. Application of precise N to the GPS heights would probably improve the comparison.

Nevertheless, the absolute average difference of ±19mm is satisfactory for most hydraulic design unless the terrain is very flat. If terrain is flat, the GPS heights in turn may be expected to be relatively free of undulation errors and thus more accurate. Table 13 indicates that the average difference in the Western Sydney plains is ±12mm, with a maximum error of 35mm.

Provided poor observing conditions, trees, close buildings etc are controlled by, for example, occupation of offset stations, AHD height accuracy of better than 20mm appears reasonable.

A further semi-absolute measure of height may be found by comparing network adjustment height control values, usually 2nd Order SCIMS values, with the values for the same control stations when unconstrained in the network adjustment. Here the relationship of vertical control to the best fit ellipsoid through this control is displayed. COMPNET displays such differences as part of the geometric contouring module, a summary of such comparisons for eleven WB series adjustments being found in Table 14.

WB	AREA	CONTROL STNS	|AV| mm
125	BLACKHEATH	7	13
127	PENRITH	11	8
129	RIVERSTONE	8	15
130	ST MARYS	8	7
139	PARRAMATTA	12	6
140	HAZELBROOK	17	11
144	SILVERWATER	9	10
145	NEUTRAL BAY	6	17
148	BUNDEENA	8	7
150	COBBITTY/MENANGLE	22	15
151	WALLACIA/COBBITTY	18	16
		|AV|	11

Table 14

Again the agreement with control (absolute average correction) is consistently less than 20mm, with an overall average of 11mm.

7. CONCLUSION

The quality of AHD heights from Rapid Static GPS baselines is influenced by the following factors...

- Number and distribution of control stations.
- Length of baselines.
- Observation/reoccupation regime.

- Signal impediments - tree cover and multipath.
- Terrain and geology - ie geoid undulations.
- AHD conversion method.

If observing conditions are unfavourable, significant tree cover or possible multipath for example, unmodelled systematic errors, possibly greater than 50mm, may be present in a height determined from a single site occupation. In such cases reoccupation in a different satellite window is prudent.

However, direct measurements generally suggest that for baselines of less than 5km, with an unobstructed sky window and the influence of low elevation satellites monitored, height accuracy of 20mm or better is achievable.

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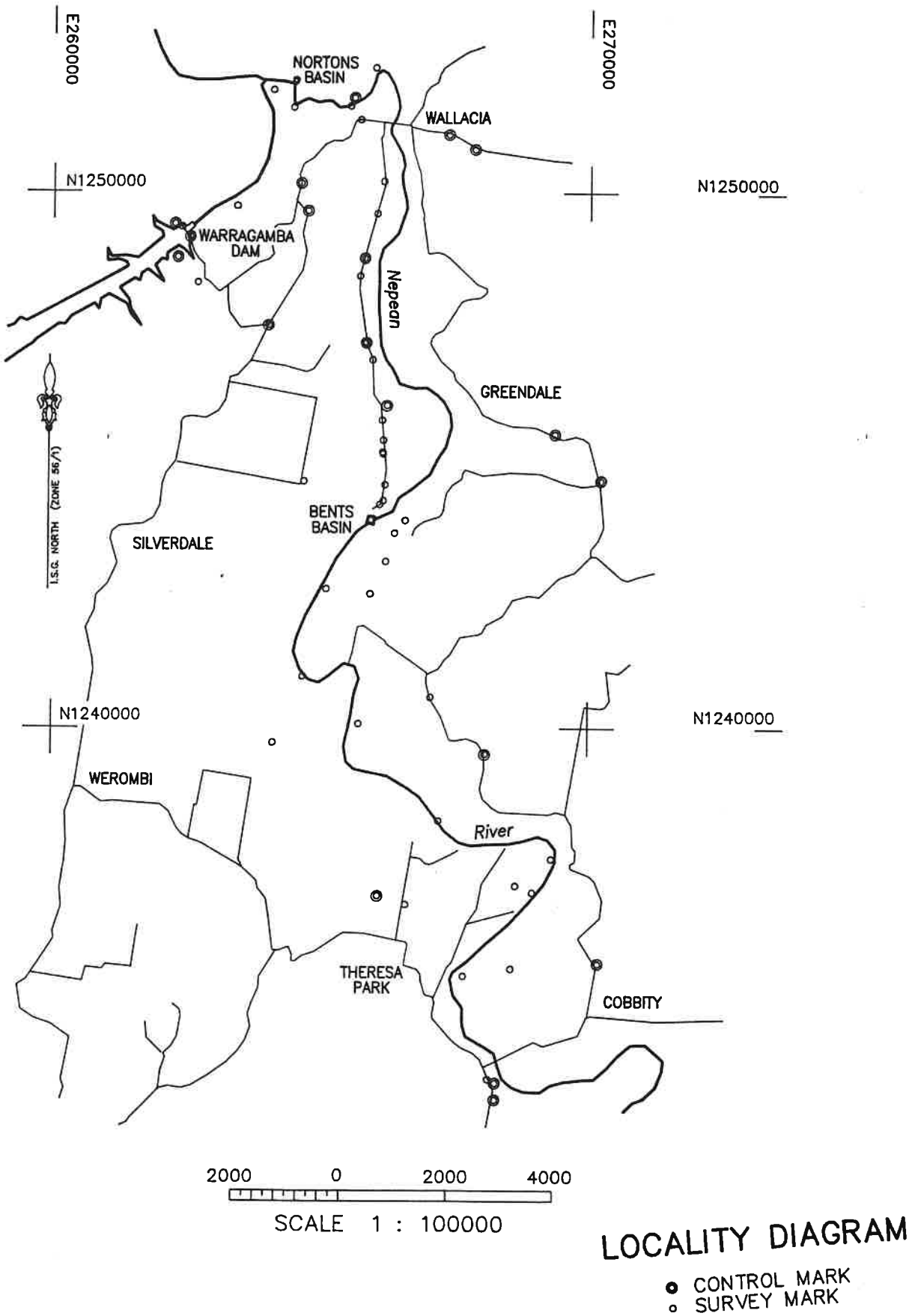
Figure 1: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E270000
NORTONS BASIN
WALLACIA
N1250000
N1250000
WARRAGAMBA DAM
Nepean
GREENDALE
I.S.G. NORTH (ZONE 56/1)
BENTS BASIN
SILVERDALE
N1240000
N1240000
WEROMBI
River
THERESA PARK
COBBITY
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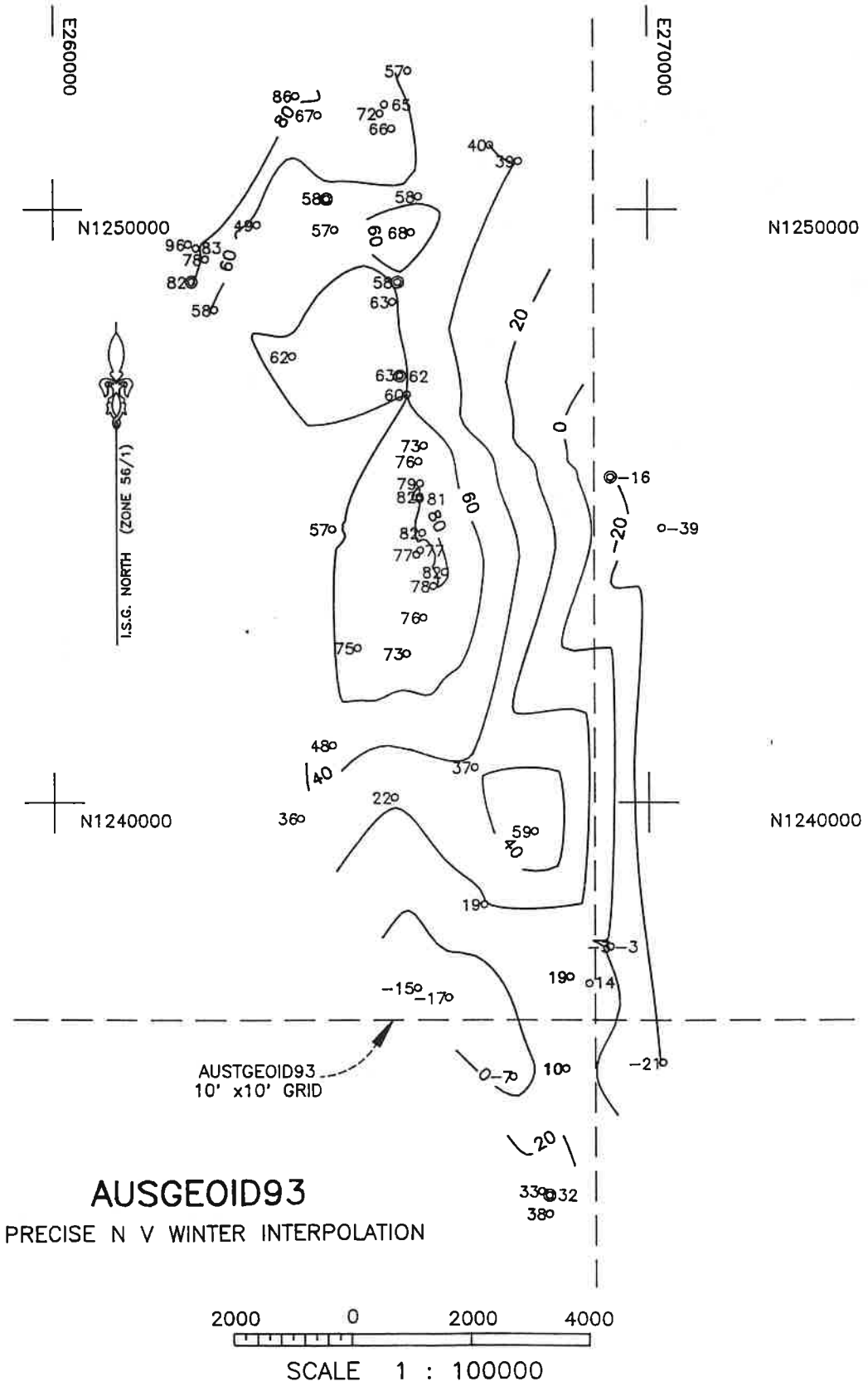
SCALE 1 : 100000
LOCALITY DIAGRAM
CONTROL MARK
SURVEY MARK

Figure 2: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E270000
N1250000
N1240000
I.S.G. NORTH (ZONE 56/1)
AUSTGEOID93
10' x10' GRID
AUSGEOID93
PRECISE N V WINTER INTERPOLATION
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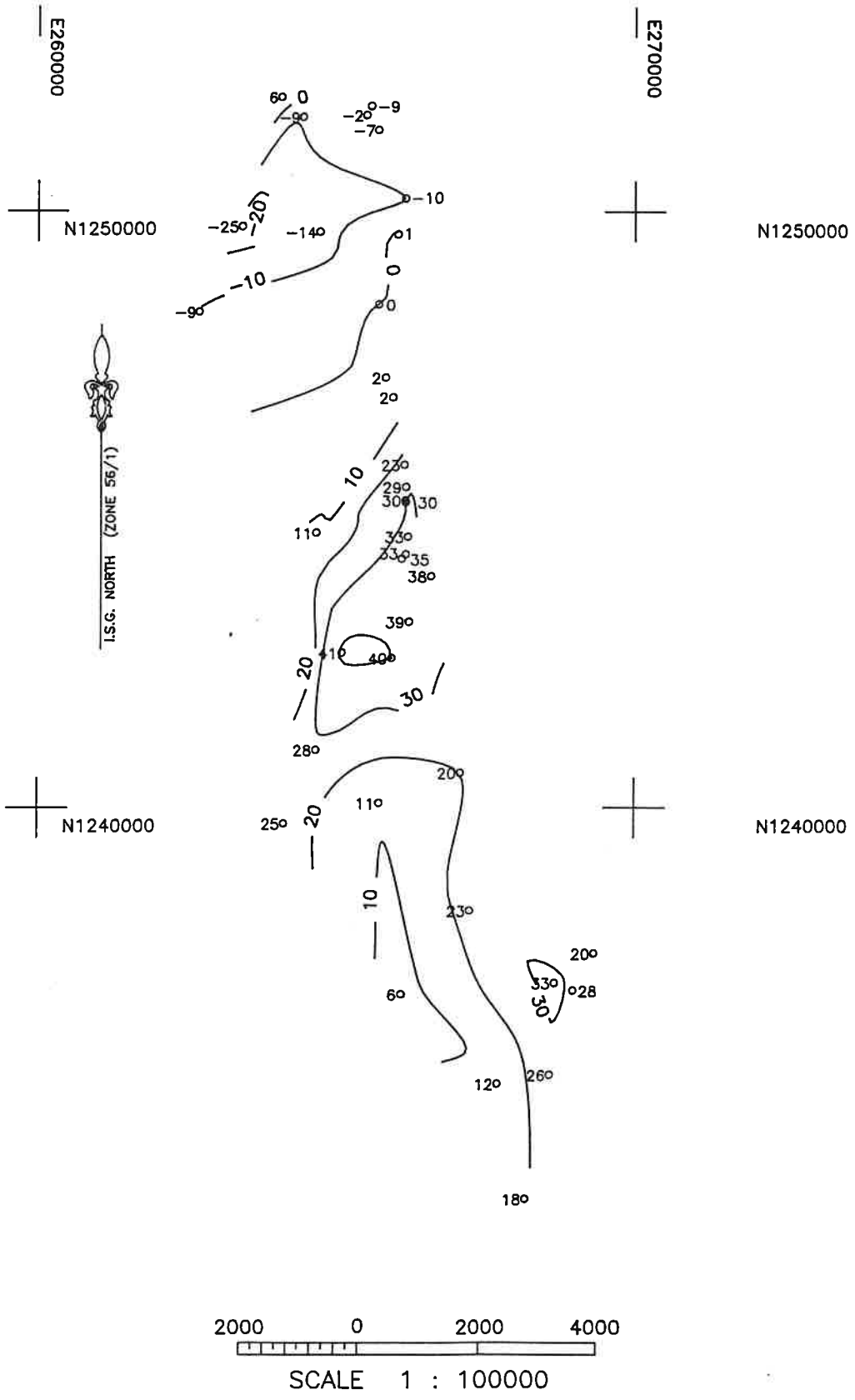
SCALE 1 : 100000

Figure 3: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I.S.G. NORTH (ZONE 56/1)
2000 0 2000 4000
SCALE 1 : 100000
GEOMETRIC V PRECISE N